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THE VALUE OF IMPROVED (ERS) INFORMATION
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DOMESTIC DISTRIBUTION EFFECTS OF U.S.
AGRICULTURE CROPS

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THE VALUE OF IMPROVED (ERS) INFORMATION BASED ON DOMESTIC DISTRIBUTION EFFECTS OF U.S. AGRICULTURE CROPS

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NOTE OF TRANSMITTAL

This report is prepared for the Office of Application under Mod. 2 of Contract NASW-2558. It represents an investigation of the value of improved (ERS) information by empirically estimating the effects of such improved information on crop inventory holding for U.S. Domestic consumption of wheat.

New estimates of a U.S. demand function for wheat and a cost of wheat storage function are developed. Wheat spot and futures markets were simulated using Monte Carlo techniques. A new theoretical model of market determinations of wheat equilibrium is calculated from empirically estimated parameters as a function of harvest forecasts.

These advances in the state of the art of measuring the value of improved information make it possible, for the first time, to authoritatively determine the value of ERS information to the U.S. wheat economy.

This is done in this report. In doing so we went substantially beyond the normal requirements of performance.

ABSTRACT

This report describes the results of an investigation of the value of improving information for forecasting future crop harvests. The study is part of a larger effort to evaluate an information gathering system based on remote sensing using satellites orbiting the earth. However, the theory and measurement methods developed in this study are not dependent upon the detailed features of the information system. Primary emphasis has been placed upon establishing practical evaluation procedures of general applicability, firmly based in economic theory. The first five sections of the study are devoted to this. We believe the greater part of the theory developed is new.

Since practical applicability was an important criterion guiding our work we devoted the greater part of our effort, in terms of time at least, to implementing the analysis for the case of U.S. domestic wheat comsumption. This involved new estimates of a demand function for wheat and of a cost of storage function. As far as we know these represent a very significant improvement, in terms of econometric techniques upon studies available in the literature.

Another important component of the implementation effort was a Monte Carlo simulation of the wheat spot and futures markets. Since inventory adjustment is the point at which information is used in the analysis, it was necessary to have a model of market determinations of wheat inventories. Market equilibrium could be calculated

from the empirically estimated parameters as a function of forecast harvests only if the carry-over horizon is known. That is the date in the future at which it is expected that the inventories of the grain in question will be completely depleted, normally the point at which the flow of newly harvested grain is beginning to swell in June. In our theoretical analysis we showed how this horizon could be determined by the solution to a certain non-linear programming problem, the parameters of which include the forecast harvest levels, which are random variables. To obtain the distribution of carry-over horizons from postulated distribution of forecasts by analytic methods is not feasible, and hence the operation of the wheat market was simulated, computing the carry-over horizon as well as such related variables as spot and futures price at each stage. The model is easily adaptable to other markets. We are not aware of any similar study in the literature.

The empirical pieces of the study are put together in section 6. The results are shown to depend critically on the accuracy of current and proposed measurement techniques. Surprisingly, these pieces of information were not readily available. While it may be that further search of government agency sources will fill this gap, the quantitative results at this stage are best presented parametrically, in terms of various possible values of current and future accuracies.

"Accuracy" can be described by a 95% confidence interval. Accuracy in measurement of such variables as acres planted in a crop translates into accuracy of the forecast relative to what it would

be if the planted acreage were known perfectly. The following table given in column (2) the estimated loss to the economy associated with a 95% confidence interval about the "true forecast" of annual wheat harvest, measured as plus or minus the percentage in column (1):

Annual Loss to the Economy due to Measurement Error

(1) 95 Confidence Interval for Annual Crop Measurement Error	(2) Annual Loss in Millions of 1973 dollars
± 1%	3.4
± 2	13.6
± 3	30.7
± 4	54.5
± 5	85.2
± 6	122.6
± 7	166.9

Note that the confidence interval in column (1) of the table should not be equated with two standard deviations of forecast error, since the latter is a compound of measurement error and variability due to weather, pests, etc. While statistics are plentiful on crop forecast error, data on measurement error have proved elusive. One bit of evidence did seem to refer to the desired quantity, placing the "average smaple error" at 2.1%. If we interpret ± 1.96 times 2.1 as the boundaries of the 95% confidence interval we obtain as estimated annual loss of 15.02 million

dollars (3rd quarter 1973 dollars). Cutting this error in half would * generate a gain equivalent to 11.4 million dollars per year. The value of reducing the measurement error as promised by an ERS space system, and its sensitivity to changes in critical paramters is shown in Table 1.1.

It is emphasized in the study that the results of the model are illustrative only since the loss estimates are sensitvie to the measurement error, for which no adequate estimate is available. The parametric approach to using the model in relation to measurement error assumptions is therefore recommended.

The theoretical model developed in the study makes it possible, as well, to calculate the value of increased <u>speed of availability</u> of information. Obtaining information with a shorter lag is tantamount to obtaining more accurate information, since the naturally occurring random events introduce a discrepancy between past values of variables composing a forecast and the present values upon which the theoretically ideal forecast would be based. Preliminary work suggests that for the case of wheat, reducing this lag by one month may be worth as much to the economy as eliminating all measurement error without reducing the lag. While the calculation procedures have been worked out, however, as of the time of submission the required programming had not been completed to apply them.

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* D.B. Wood, et. al., "The Use of the Earth Resources Technology Satellite (ERTS) For Crop Production Forecasts", Draft of Final Report, Goddard Space Flight Center, July 24, 1974

Table 1.1 The Value of Reducing Measurement Error Based on Goddard Task Force Results on ERTS and Earth Sat (Millions of 4th qtr 1973 dollars annually)

Price Elasticity [d] for Wheat Demand	The Measurement Error at Completed Harvest (Annual)			
	2.2% [c] (Winter Wheat)	2.5% [a] (Winter Wheat)	3.3% [a] (Spring Wheat)	4.4% [c] (Spring Wheat)
-.10 [a]	62.4	80.6	140.5	249.8
-.50 [b]	12.5	16.1	28.1	50.0

[a] This value was quoted in the Earth Sat case study on agriculture

[b] The basic estimate used in this report

[c] Goddard Task Force Results on ERTS

[d] Based on United States domestic use of all wheat

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Introduction

In this report we develop the theory necessary to evaluate improvements in the measuring system used to produce grain crop harvest forecasts. Crop forecasts are used by a variety of agents in an economy for consumption and production planning. We singled out two classes of agents of particular importance: farmers (in their planting decisions process) and inventory holders (in determining how much to hold). Of these, in turn, we argue the uses of better information by the second group are likely to generate the larger share of benefits. In addition, it turns out that the way in which a theory of inventory determination leads to a value of information is somewhat simpler than that required to incorporate producer decisions. Accordingly, deciding in favor a greater depth of analysis over greater breadth at this point, we decided to consider only the benefits derived from improved inventory decisions.

This is not the same thing as considering only benefits to inventory holders. Quite the contrary is the case of the economic system we study most closely, the competitive market system. The tendency of competition to eliminate super-normal profits causes the benefits of improved information to be transmitted to those selling to and

buying from inventory holders, namely farmers and consumers of wheat.

Actually, very little grain can be said to be consumed "directly", since milling and baking are necessary to produce bread, breakfast cereal, noodles, etc. The use of grain as an input to some further production process is considered to be "consumption", as distinguished from storage. Since the demanders of wheat from the inventory system include such producers, some of what we label "consumption benefits" will actually occur in the form of increased producers' surpluses (rents), although, again, in a market system competition tends to lead to a further passing along of such gains to ultimate consumers.

The "objective" form of the benefits derivable from better information is taken to be a smoothing of the flow of consumption. (In a market system this corresponds to more stable prices.) The value attributable to reduced variability of the grain consumption flow dervies from the pehnomenon of diminishing marginal valuation, the tendency for increments of a good to be more highly valued when little is available, and less highly valued when a great deal is available.

Although there is a world grain market, and our theorectical model applies as well to that system as to a

single national market, in applying our analysis we chose to confine attention to the benefits generated for U.S. residents arising from improvements in forecasting U.S. harvests of wheat. (Note that one could sensibly consider the benefits generated for world residents from better forecasting of U.S. wheat harvests, or benefits for U.S. residents from better forecasting of world wheat harvests. The same methods apply, although different econometric problems would be encountered.) The concentration on the United States was influenced in part by the obvious concern U.S. policymakers will have for benefits within the country, and in part by the availability of reasonably good data with which to estimate crucial parameters for this system.

For similar reasons, our modelling effort is directed at inventory determination in a market system. Crop forecasts are, obviously, produced and used in economies organized in other ways. Indeed, the active intervention of the U.S. government in the domestic market system means that even in the United States the market model has not been the appropriate one for many periods. However, at present the competitive market mechanism dominates the determination of grain inventories in the United States. This is fortunate, since modeling the political determination of inventories poses more difficult problems.

Section-by-Section Summary

The layman understands well that information can be valuable, but that the value to one agent may be at the cost of value to another. The football defense based on a knowledge of the other team's signals is sure to be a good one, but that gain due to better information comes at the expense of the offense. On the other hand, some information, such as the timing of the crest of a flood, is clearly of general social value. In section 1 of the report we present an informal discussion of the value of more accurate crop forecasts, attempting to isolate the concepts which we subsequently incorporate to the formal model.

The "better" information obtained by advanced technology methods is not itself in the form of better forecasts. The remote sensing devices and associated information processing systems produce improved accuracy of measurement of such phenomena as planted acreage, crop growth rates, etc. This information is used to produce forecasts by incorporation into a forecasting model. There is a tendency to equate shortcomings of forecasts with shortcomings of information, but the first may arise through bad forecasting models and through the sheer randomness of events occurring through the time between forecast and outcome. Section 2 describes the model of crop forecasting used in this study. The notion of "ideal forecast" at a point in time is introduced. This is the forecast which could be constructed on the basis of

perfect information about the things which are knowable at that time. The measurement error component of a forecast is assumed to arise from imperfect perception of ideal forecasts. Measurement improvements result in better estimates of ideal forecasts.

Information may be improved in another way as well, by reducing the lag between the date of measurement and the availability of the resulting information in the form of a forecast. The framework established in section 2 makes it easy to keep track of this aspect of information quality, which seems likely to be an important one in the application to satellite systems.

In section 3 we show the way in which better information converted into improved forecasts can lead to improved inventory decisions. The important point is established that the value of information depends upon the rule or procedure by which it is built into decisions. If the use of information is not appropriate, "improved" information may be valueless. Using a one-person, Robinson Crusoe world, we develop a measure of the value of information and a theory of Crusoe's incroporation of information to his inventory decision.

Crusoe is modeled as solving an optimization problem. With only minor modification, the general form of his problem can be used to describe that of inventory determination

in a market system. Whereas we could simply assume an objective function for Crusoe depending upon his monthly gain consumption, it is necessary to derive a social objective function. We use the area under the demand curve to represent the dollar value of any specified quantity of grain consumed. The benefits of an improved information system are taken to be measured by the expected value of annual grain consumption (by month) less storage costs. This is set out in section 4.

While Crusoe's inventory decision could be derived from his optimizing behavior, the rule by which forecasts influence inventories in a market system must be determined from the profit-maximizing behavior of many competitive inventory holders. The profits of competitive inventory holders occur in the form of the capital gains on their stocks. If the increased in price from period to period is large enough to compensate for storage and interest costs they hold addition inventories. If the price increase expected is too little, inventory holders sell off their stocks. The price is determined by the amount made available to consumers, which is the sum of heldover inventories and current-period harvests, less inventories carred forward. Thus, in order to predict prices, inventory holders must predict their own future decisions. In section 5 the way in which this system can be closed is derived. Along the way, futures markets are

introduced to coordinate the expectations of inventory holders as a group.

By the end of section 5 we have a full theory of the relationship between information as translated into forecasts and competitive inventories. Section 6 puts all of the pieces together in an empirical application, calculating the value of improved information in the case of the U.S. wheat market. We adduce functional forms and parameters to the key relationships of the model, and carry out the calculations. Most of the required parameters can be estimated with reasonable confidence. An exception is the current and prospective degree of accuracy of measurement systems. The final estimated results are therefore presented in parametric form. For those interested simply in the numerical results, Figure 6.1 and 6.2 shows our best estimate of the value of introducing an ERS space system based on the Task Force Report results on ERS.* On the basis of that evidence, we can guess that the current levels of accuracy allow us to come within plus or minus 7.6% of the ideal monthly forecast harvest for any month about 95% of the time. If we reduce this confidence interval to plus or minus 3.8%, the estimated gain to the economy is equivalent to 11.4 million (4th qts. 1973) dollars per year.

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* D. B. Wood, et. al., "The Use of the Earth Resources Technology Satellite (ERTS) for Crop Production Forecasts", Draft Final Report, Goddard Space Flight Center, July 24, 1974.

Table 6.5 indicates how this particular measurement improvement would be affected by various changes in the underlying parameters. Although the range noted there is large, this is the result of including for comparison purposes a parameter value used in other studies, that of the elasticity of demand, which we have replaced by new econometric work. In fact, our estimates of demand elasticities appear to be a great improvement upon those available in the literature, and they seem to be robust to changes in the specification of the demand model. Hence one can with some confidence place the gain from the specified information improvement in the range of values shown in Table 6.5.

While we feel some confience in the numerical results presented by section 6, it should be kept in mind that our major objective was to produce evaluation procedures of general applicability, firmly based on economic theory. The main "product" of the study is the procedures themselves. Parametrically, these are best demonstrated by a graph of economic loss caused by wrong inventory decision against measurement error as in Figure 6.2.

Section 7 presents suggestions for further work in the contest of a review of the main links in the chain of

reasoning. In fact, the very last subsection of the main text contains a summary of the model which may be profitably read as introductory material.

1. Informal Discussion of the Value of Accurate Forecasts

The subject of the value of information is a broad one and it will be useful to keep in min that the information of which we speak concerns the current value of certain measurable quantities. The devices under consideration are expected to provide accurate information about the current status of different agricultural crops, which will enable us to predict with greater accuracy than with current methods the quantity of those crops which will emerge from the farm at specified times in the future. Information of this kind may be distinguished at least for practical purposes from information about new technologies, which in principle might never be revealed at all.

A forecast of the future is expressable, explicitly or implicitly in the form of a probability distribution. Such a distribution may be though of as representing the degree of certainty of a person's beliefs about the future. For example, we may say of the particular day July 4, 1974, that it will rain on that day with probability .3 and it will not rain with probability .7. As the day comes closer it may become possible to dicover by meteorological analysis that July 1974 is going to be a particularly rainy month and therefore we revise our estimate, increasing in our minds the probability of rain. At one minute to midnight of July 3, 1974 we may be able to state with a very high degree of condifence whether it is going to rain or not, in which case our

belief would be expressed in the form of a probability 1 of that event which we by that time consider most likely.

Of course, a forecast is usually summarized by a single number: the wheat harvest forecast for the year 1975 will be a number such as 2,000 million bushels. This number is the mean of the distribution of harvests characterizing the belief of the person making the forecast. Equivalently, we may think of the beliefs as characterized by this number plus a distribution of errors, the various deviations between the 2,000 bushels forecast and what the forecaster anticipates will actually occur. Corresponding to this subjective distribution is an observable (in principle) distribution of deviations between the forecast and what is known to have occurred after the fact. These observable quantities are what are normally referred to as "forecast error." We note that the subjective distribution is the one relevant for decision-making. For the most part we shall use the term forecast error to refer to both concepts, referring to the distinction only where confusion may otherwise result. We shall assume that such distributions are completely determined by specification of mean and variance; sometimes we shall treat them as Gaussian normal.

Forecasting error variance expresses our degree of uncertainty, which may arise from two sorts of sources. First, we may not have a very good idea of what the state of the world

is now or has been in the past. For example, we may have only a crude thermometer available to assist us to forecast the afternoon temperature. Second, there may be events which are genuinely random, or may be treated as such, which will occur between now and the time point to which our forecast is directed, which make it impossible for us to know the future with certainty, no matter how clear our picture of the present state of affairs: no matter how accurate our knowledge of the starting point of the roulette ball, we may not be able to narrow the forecast error on its ultimate stopping point. (The example illustrates the ambiguity of the distinction. Presumably if we <u>really</u> understood the roulette wheel and could calculate well enough, we could improve our forecast.) The "information" we shall be discussing here is directed toward reducing the variance due to the first source. Improved information allows us to make more accurate forecasts, expressable as a reduction in the dispersion of our subjective distribution of forecast quantities before the fact and, correspondingly, a reduction in the dispersion of the experienced forecast error (deviation between forecast and actual quantities) after the fact. Such a reduction might be achieved by obtaining from the farmer precise information about the amount of wheat he plans to plant in June 1974. While, before the harvest, the uncertainty about the outcome resulting from weather variability remains, the information about the planting allows

us to construct a guess about the resulting outcome in September which is more accurate than the guess in the absence of the information. The degree to which our estimate is improved can be expressed by a reduction in the variance of the subjective distribution and of the forecast errors.

The value of information thus depends upon the value of good forecasts. In the remainder of this section we discuss in an informal way why forecasts are valuable, and to whom. This will form the basis for our subsequent formal theory and measurement.

The Meaning of the Value of Forecasts

When we speak of the value of forecasts we must distinguish carefully between value to the entire economy and the value to a single individual. As is well known, it is often possible for an individual to reap large gains from a possession of knowledge of great accuracy or at least possession of knowledge of greater accuracy than that possessed by others. We may illustrate this by the example of a price prediction, let us say of a painting by Rembrandt which is to come up at auction in September 1974, and which is now on the market for purchase in January 1974. Knowing exactly what the Rembrandt will sell for 8 months hence, I can make a certain decision now what price it is worth paying. The accurate

forecast of the future allows me to make with certainty a gain now. Note, however, that should the information lead me to decide to buy the painting now, in January, the effect is to shift to me the profit obtained by the difference between the selling price now and that 8 months hence, but at the cost of an equivalent gain in the hand of someone else who might have purchased the painting if I did not. The opportunity would obviously have been lost to me were the information I possessed about the price to rule in September available generally instead of available to me alone.*

In this illustrative case we see that the sole effect of improved information in the hands of a single individual is to alter the incidence of a gain from one person to another. Presumably the ultimate purchaser of the Rembrandt in September would have ended up holding the painting in any case, and the only effect of improved information is to place the gain in my hands rather than in someone else's hands. It is usual in applied welfare economics, although not always justifiable, to equate equivalent dollar amounts of gains by one person

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*It would be desirable to have different terms for the various meanings of the word "information" occuring in this study. Strictly speaking, we intend the word to refer to an estimate of some observable quantity, such as the number of acres planted in wheat. In this sense, a forecast is not "information", at least given the current development of normal human perceptions. It seems rather pedantic, however, to enforce this distinction throughout the text, and we believe no confusion will result from our usage.

with the same amount of gains by someone else. In this case we would say that there has been a private gain to me offset by an equivalent loss to someone else from the improved information about the price of the Rembrandt in September 1974. Although there have been possilby large changes in private wealth as a result of this information we would say, loosely speaking, there is no social gain whatsoever.

This distinction between private gain and social gain may be even more dramatically illustrated by pointing out the possible advantage to an individual of misinformation in the hands of others. Thus, if I wish to purchase a piece of property it may be greatly to my advantage that everyone else in the world thinks it highly likely that a major highway is going to be built across that property, even though I know with certainty that this is not the case. Even though the misinformation may lead other people to make bad allocative choices, I stand potentially to make a substantial gain. Again the crucial point for estimating private gain is the degree of inequality or asymmetry of information in the hands of different agents. In this illustrative case it should be clear that there is no social gain in the usual sense to be had from the promulgation of misinformation, even though this might be greatly to one individual's private advantage.

Sources of Social Gain from Improved Forecasts

There are two broad sorts of social gain from a general reduction in crop forecasting error. First, by virtue of good forecasts of forthcoming corps a society is able to make improved allocative decisions. Both by making better timed dispositions of inventories of available farm products, and by making planting decisions in better anticipation of the total crop harvest, the society can optimize the flow of consumption over time. The underlying idea is that it is desirable to have a smooth flow of consumption of commodities, rather than an irregular one. This is the familiar principle that the value of increments to consumption of a good decreases as the quantity consumed increases: The value of an additional bushel of tomatoes in the presence of a large crop in August is much smaller than the value of an increment of a bushel in the middle of winter when few tomatoes are available.

Secondly, a reduction in the dispersion of the subjective distribution of forecast errors, i.e., an increase in the degree of certainty, may be valued in itself. We customarily assume that economic agents prefer a certain outcome to situations in which the average of expected outcome is the same but with some variance. It is this value which is referred to when we speak of individuals having risk aversion, the prevalence of which is suggested by such phenomena as insurance and portfolio diversification.

In this study we consider only the gain of the first sort, that arising from our ability to make decisions which are less likely after the fact to have proved incorrect. The value of reduced uncertainty per se will be ignored. In the context of the models of behavior of agents in markets under uncertainty which follow, the assumption that uncertainty per se is not a source of loss of value will be reflected in the assumption that agents act to maximize expected monetary profits.

This Study Concentrates on Inventory Adjustment Gains

Within the class of allocative gains we shall further restrict our attention to those resulting from improved inventory choices. There are two reasons for this. The first is that in the case of wheat, the crop to which our analysis will be applied empirically, the possibility for significantly adjusting production within the crop year appears limited. This means that we are guessing that the size of the gain from this source is small relative to that available from the inventory improvements. It would, no doubt, be most desirable to test this guess by carrying out the analysis and measurements, which brings us to the second reason for starting with a concentration on inventories. As we shall see in the succeeding sections, the analysis of this problem is simpler than that of the case of endogenous supply decisions. Since the chain

of reasoning and calculations we shall be tracing is already rather long, there is a great advantage in resisting the further complication. At the same time, while our expectation was fulfilled that it is possible to obtain highly convincing econometric estimates of demand parameters, there is every reason to expect great difficulty in estimating supply parameters. Thus, both reasons of theoretical complexity and estimation problems reinforce our preference on ground of expected relative potential gain for considering the pure inventory adjustment model.

The Distribution of Gains from Improved Information

It may be thought that the gainers and losers from the production of new and better information are affected by the way in which the new information is introducted into the system, and this indeed appears to be the case. As the example of the Rembrandt auction suggests, particular agents to whom new information is first communicated may be able to reap large personal benefits at the cost of benefits to others. The importance of dissemination procedures is well recognized in, for example, the regulation of "insiders" in security markets.

An example might be made of a discovery by a cooperation of large deposits of some mineral. This discovery will be reported to the general public on a specified data in the future; in the meantime it is of extraordinary value to an insider who

may be able to capture enormous speculative gains, much as our Rembrandt purchaser was able to in the earlier illustration. By the same token it is clear that it is possible to introduce information in some ways which is actually harmful to individuals, at least in the *ex post* sense. In this case, for example, the individual who sells his stock in the company which has discovered the large mineral deposit will certainly after the fact be less well off than he would have been had all of the information become available on the date in the future when it was to be made generally public.

Of course even information in the hands of a stock market insider is transmitted at least partially to the general public via the very process by which that individual capitalizes on his advantage, in this case through the resulting increase in the price of the stock of the corporation in question due to his purchases. In this way information in the hands of the insider is related to decisions of other people by their observation of the market price of the stock.

Similarly in the case of improved forecasting, the potential speculative gains accruing to individuals in possession of improved information are obvious distributional consequences; since these gains must be balanced by losses of those who do not have access to the improved crop information, the result is shifting gains from one group to another. Here too, no matter where it is initially introduced, the information would in part be made available to the general

public, at least in its crucial aspects, via the movements in price which would be generated by its possession in a single individual's hands. Just as in the case of the Rembrandt painting, however, the speculative gains may be entirely offset by speculative losses and the net social benefit might be zero or very small. The implications for social policy of the precise method releasing information therefore appear nontrivial.

At the opposite pole from the stock market insider archetype is the government statistical information made available in a carefully controlled way to an entire group of people at once. The ideal picture of this sort of information release is a report on our corporation with the large new mineral deposit appearing for the first time in a Sunday newspaper on which day the market in which the company's stock is traded is closed. Now we have no price changes occuring during a period in which information is asymmetrically distributed. Rather, the market opens on Monday morning with all of the agents in possession of the same new knowledge Who gains and who loses? Paradoxically, in ex post facto sense, it would appear that there do exist possible losers from introduction of better information. Let us suppose, for example, that the information is an increase in the forthcoming supply of some crop. As a holder of the stock of this commodity I had planned on Monday morning to sell my entire inventory on the market. The new information will cause the market price of this commodity to decline and I will therefore have

been made worse off by its introduction. Again, for every such loser there is a corresponding gainer, and it is difficult to make a strong case for a particular distribution of such gains and losses without going into considerable greater detail along normative lines. There seems to be some normative advantage to reducing gains and losses attributable to asymmetrical information, but it is not entirely clear that this is well grounded.

If we consider a more prior sense of gains and losses, and imagine that we can all choose whether the government should make available at some date in the future a particular report about forthcoming crops, we expect intuitively a preference for the system where this report is made. (Counter cases could be constructed, however.) On the other hand, if we imagine that the crop information is going to be made available to an insider, it is not at all hard to imagine our wishing rather that the information not be made available at all. It might be fruitful to examine in greater detail the difference between these two cases.

There is one important group of people who would be averse to the government's introducing a new statistical service, for example, and these are the people now engaged in producing information and marketing it. Obviously such information producers are potentially hurt by the introduction of a new information source.

2. The Model of Forecasting Used in This Study

The construction of a forecast involves two main elements: information about what is the current status of various features of the world and a model of how the currently observable features influence the variable being forecast. Suppose, to pose an illustrative example, we are interested in knowing into which of seven holes a pinball will roll at the end of its run down an inclined plane studded with the usual obstacles. Let us consider how a forecast is constructed.

We start with a model of how the ball will roll starting from a given point with a given velocity. This model consists of the laws of motion and of knowledge about the positions and physical characteristics of the obstacles, by which it is possible to compute the path of the ball. Typically there will be unknown or imperfectly known elements of the physical system. Furthermore shocks may be anticipated from outside of the system which will influence the path of the ball; the pinball machine may be located just above a subway tunnel. As a result, even if we know the starting point, our physical model of the system does not generally allow us to predict exactly the path of the ball. We might typically express our forecast of the final location of the ball in the form of a single number (e.g., "hole number 3"), but this normally is simply the central tendency of an implied probability distribution.

If we have precise knowledge of the position, direction of motion, velocity of the ball at a given point in time we can predict its position at any later time using this physical model, which is what we referred to above as a model of how the currently observable features (position, direction and velocity) influence the variable (future position of the ball) being forecast. Because of what may be regarded as truly random aspects of the system within which the ball is moving, our forecast must be itself in the form of a probability distribution, even though we may express it in the form of a single number. Furthermore, because of the cumulative nature of the random shocks through time, the dispersion of our forecast distribution of the positions of the ball is likely to increase as the distance into the future over which we are attempting to forecast increases. In looser and more commonplace language, long-term forecasts are "less accurate" than short-term forecasts owing to the greater intervention of random influences.

As was suggested above, there is in addition to nature's randomness, another source of "inaccuracy" of forecasts, associated with inadequacy of <u>information</u> about the current state of the system, in this case the current position and velocity of the ball. Let us suppose, for example, that these are obtained by the observer using a ruler on top of the glass cover of the pin-ball run and a stop watch. Assuming that the observer is capable of instantaneous calculation of the forecast

once he is given position and velocity, he can convert his observation of these variables into a prediction at once. However, the procedure for obtaining position and velocity is itself subject to error, which we shall refer to as sampling error or measurement error. Measurement error would cause forecasts to be random variables, with some degree of dispersion, even if the model of the system were perfect and the system itself not subject to outside shocks. The dispersion or inaccuracy of actual forecasts is thus a compound of nature's randomness and measurement error.

This study is primarily concerned with the value of reducing the measurement error in the construction of crop forecasts. It is clear that this is only a part of the source of dispersion in crop forecasts. However, even though variability due to nature's randomness is great, and there is correspondingly a large potential for improving forecasts by improvements in the model of the crop production system (e.g., by deeper understanding of the determinants of weather), we shall see that relatively small measurement errors are surprisingly costly. As a result there are substantial gains to be made by improving the information about the current state of the system, i.e., by reducing the measurement error.

There is a further way in which information can be improved. This is the reduction in time between the observation or measurement of the state of the system and the

<u>availability</u> of the information for use in the form of a forecast. Such a reduction seems particularly likely in shifting from methods of sampling involving postal or telephonic communication of observations to a central calculating unit -- as when field units report to the U.S.D.A. -- by an advanced technology method based on satellite observation, in which information is handled electronically as a matter of course at every stage.

We refer to the time elapsed between the actual observation of the state of the system and the production and transmission of a useful forecast based on that information as the <u>availability lag</u> associated with a forecasting procedure. This may be illustrated with our pinball machine. Suppose that the initial procedure involves measurements, using the ruler and stop-watch, which are then entered into a mechanical calculating machine to produce a forecast of the path of the ball. Because the calculations take time, by the time a forecast has been made the ball is no longer at the point on which the forecast is based. The forecast, in other words, is constructed on data about the ball at some time in the past. The longer is the lag the less useful is the forecast for two reasons. First, the longer the time which has elapsed, the less useful is the historical position and velocity of the ball as a predictor of its current position, because the ball has in the meantime been subject to

nature's random shocks. Second, the longer is the delay, the less remains of the ball's path to be predicted. If the delay is long enough, the forecast arrives after the ball has already reached the end of its run! The forecast is then of use only in checking the adequacy of the model of the system. It arrives too late to help the person wanting to place a bet on the final position of the ball.

The two aspectes of improving the information base for forecasting are thus interrelated. The shorter is the availability lag the more valuable is any given reduction in measurement error.

A rough analogy exists between the pinball forecasting problem and the idealized version of crop forecasting used in this paper. We take time to be broken into discrete months. The problem of crop forecasting is not to follow a single ball through time but rather several balls in the form of monthly harvests. Let G_t (sometimes we shall write this equivalently as $G(t)$) denote the quantity of the grain of interest harvested during month t. This notation will be used throughout, although later, when exports are introduced we shall let G_t stand for "effective harvest", or actual harvest less exports.

It is assumed that, on the basis of perfect information about conditions on the ground, numbers of acres planted in the specified grain in each of several geographical regions,

visible condition of ripeness, etc., forecasts can be constructed of the quantities to be harvested for each of a certain succession of coming months, using a model of how grains evolve over time as they mature. Such forecasts are subject to error due to nature's randomness. We speak of this <u>set</u> of ideal forecasts, which would be made in a given month on the basis of perfect information about what is in principle knowable in that month, as the <u>state of the system</u>. The state of the system as of period t is denoted by S_t. S_t is a <u>vector</u> of <u>ideal forecasts</u>; its first component is S_t^{t+1}, a forecast of G_{t+1}, etc.:

(2.1) $$S_t = (S_t^t, S_t^{t+1}, \ldots S_t^{t+M+1}).$$

Note that the superscript which identifies a component of S_t identifies the period for which an ideal forecast is being made.

Actual forecasts of crops are based not upon perfect information but upon measurements and samples of such quantities as acres under cultivation, height of stalks, etc. These are subject to sampling or measurement error. When the data are fed into the model which produces forecasts, these errors result in deviations between the actual set of forecasts of

monthly harvests and the ideal set of forecasts represented by S_t. Great simplification in our analysis is effected by regarding S_t *itself* as the object of measurement.

It is important to be clear about this device. When we speak of sampling or measurement error, we refer to an error of measurement of S_t, *not* directly to the underlying errors of measurement of acreage, growth, etc. Since such underlying errors translate directly into errors in estimation of S_t this analytical convenience does not affect the generality of the results. However, some caution must be exercised when we come to specification of a probability distribution of percentage errors in measurement of S_t, a distribution which need not be identical to that of percentage errors in any of the components from which forecasts are calculated.

A forecast based on month t information then, is here taken to be an *estimate* of S_t. Denote by $\hat{S}_t$ such an estimate. We shall assume that the measuring devices and procedures introduce an error ψ_t such that

(2.2) $$S_t = S_t + \psi_t$$

The measurement error, ψ_t , is thus a vector, with components $(\psi_t^t , \psi_t^{t+1}, \ldots, \psi_t^{t+M+1})$.

At this point we should explain the meaning of the parameter M which occurs in the specification of S_t . We refer to this parameter as the "maturation period", a name motivated by a simple model, whereby the grain harvested in any period must have been planted exactly M periods earlier. If we take the quantity planted as exogenously given, not endogenously determined, in this model it is not possible to forecast the harvest of any month more than M periods into the future on the basis of currently observable features of the system. Of course one may construct a forecast from prior knowledge of, say, the typical periodic pattern of harvests, but this is not dependent upon an input of current information.

In fact, this simple model is only a very rough approximation to the case of wheat, the grain to which our analysis will be applied in this study. The number of months between planting and harvesting varies greatly with the type of wheat and the region of the country in which it is planted. There is no reason one could not take this into account in the model, for example, allowing M to be itself a function of t. Rather than carry along this complication, however, we have chosen to work with a constant M. It can in any case always be interpreted as the maximum number of months into the future

one can forecast harvests, with the forecast depending upon features at least in principle currently observable.

Under this interpretation we see that the last component of the measurement error vector in (2.2) ψ_t^{t+M+1}, will be identically zero. This is so because by definition of M the forecast of G_{t+M+1} cannot depend upon features observable at time t.

We have very nearly completed the description of the model of forecasting. It remains to put the availability lag back into the story. Let the symbol AL stand for availability lag. Then $\hat{S}_{t-AL}$ is the vector of forecasts available at time t. To be more precise, the components of $\hat{S}_{t-AL}$ referring to harvests occurring at or beyond month t are taken to be the forecasts available at time t. Thus, for example, $\hat{S}_{23}^{26}$ would be the forecast of G_{26} available in month 25 if the availability lag were 2.

3. A Model of the Social Gain from Improved Forecasting

It may seem obvious that more accurate and more timely information is valuable. Oddly enough this is not necessarily so. Suppose, for example it happens that a certain curative procedure followed by a physician to treat some malady is exactly wrong -- it greatly amplifies the effects of the sickness. Because it is virtually impossible to

diagonose this malady in a timely way, however, the treatment is almost never used. An improvement in the information system which produced an earlier and more accurate diagnosis will be of negative value, since the use of the information is incorrect. This simple illustration suggests how important it is, in attempting to evaluate improvements in crop forecasting information, to develop a satisfactory model of the way in which information is used.

For the various reasons indicated in Section 1, it may actually be easier to determine social value of information than its valu e in the hands of an individual who stands to gain from an asymmetrical information advantage. In this section we attempt to make precise some of the concepts involved in estimating the social gain. It is important at the outset to spell out as clearly as possible the basic ideas, and for this reason we start by confining our attention to a one-man society, a Robinson Crusoe world. We consider Crusoe's inventory problem, the problem of allocating given (but imperfectly foreeable) harvests to consumption over time.

Crusoe's problem will be constructed in such a way as to guarantee that better information is valuable. This will follow from the fact that Crusoe is explicitly attempting to optimize his grain consumption sequence, and his rules for using information are designed to contribute to this end. When we turn in the following section to the model of the use

of information in a market system we shall not have any obvious assurance that the rules by which agents in markets use information tend to be optimal from a social point of view. Thus, while there is a close analogy between the Crusoe world and a world of many agents operating in markets, there is this important difference in character between the source of the rules for using information. While it is likely that, as in many similar welfare economic models, market behavior has optimality properties, we shall not demonstrate these in this case, and whether information has positive value will have to be determined from empirical data.

Having sketched out the importance of modelling the rules of information usage and flagged the difference between the Crusoe model and the market model, let us turn to Crusoe's proble. We take Crusoe's only interest to be the consumption of two goods, an agricultural commodity, which we shall call "grain" and measure in tons, and some sort of composite of other commodities and services, which we shall call "numeraire good" and measure in real dollars (or simply "dollars" as long as we need not be concerned with price inflation).

Assume finally that Crusoe values any given amount (x) of grain consumption in one month as exactly equivalent to an extra V(x) dollars of consumption of numeraire good in that month: take away from Crusoe his x units of grain consumption in a month and substitue V(x) dollars of numeraire good consumption

and he will declare himself just satisfied with the switch.

The amount of grain consumed by Crusoe in each period depends upon the amount harvested, the amount added to current stocks, and the amount carried over from previous periods. Further in making choices about production and storage plans, Crusoe must take into account the numeraire good costs incurred in producing and storing grain.

Let Q_t be the quantity of grain placed into inventory in period t to hold over until period $t+1$, and let $TC(Q_t)$ be the total dollar cost incurred in period t to perform this storage. It seems reasonable to suppose that a certain amount of grain is lost through deterioration in storage, so let us assume that if Q_t is stored in period t then $(1-\delta)\cdot Q_t$ is actually carried forward to period $t+1$, where δ is some constant, presumably positive.

Recall that G_t stands for the output of grain from the farms in month t. Since we shall not now consider alternative plans for G_t we take it as exogenously given and ignore its cost. We can now write down the amount of grain consumed in period t as related to storage decisions in period $t-1$ and t:

$$(3.1) \qquad C_t = G_t + (1-\delta)Q_{t-1} - Q_t \ .$$

Since G_t's are taken as given, a choice of a sequence of inventory levels Q_t determines a sequence of grain consumption levels. (Of course, we cannot pick a negative inventory level, since grain cannot be moved basckward in time. Furthermore, if our sequence of inventory levels is to be feasible it must not imply a negative grain consumption level, C_t, at any time.)

Associated with a feasible sequence of inventory levels and a given sequence of grain harvests is a sequence of numeraire good values of grain consumption, from which we must net out costs of grain storage. Substituting (3.1) into $V(x)$ we can define the annual dollar value of the consumption arising from a sequence of inventories by

$$(3.2) \qquad V(G_t + (1-\delta)Q_{t-1} - Q_t) + V(G_{t+1} + (1-\delta)Q_t - Q_{t+1}) + \dots + V(G_{t+11} + (1-\delta)Q_{t+10} - Q_{t+11})$$

(Note again that this value is not defined for arbitrary sequences Q_t, G_t, since feasibility requires $Q_t \geq 0$ and $C_t \geq 0$.)

What we have accomplished thus far is to relate Crusoe's well-being to what Nature does (in the form of the G_t's) and to what Crusoe does (in the form of the Q_t's). We next consider how Crusoe picks the Q_t's and how this connects with crop forecasting.

We may presume that Crusoe makes his decision on inventory holdings on the basis of guesses about grain harvests in the future. The guesses could be completely arbitrary, but more plausibly Crusoe makes his guesses about harvests on the basis of some sort of model, explicit or implicit, of the way the future is related to the present and the past. In other words, he constructs forecasts. For example, if wheat is harvested 180 days after sowing and Crusoe knows the amount of 60 day old wheat in existence, he will forecast the wheat harvest 120 days in the future by multiplying the amount of 60-day old wheat by a factor representing typical growth rates, average losses due to insects, etc. Crusoe knows that his forecast will never be completely correct, that there will be some forecasting error, but if his guessing procedure is a good one, he will be right on average. We shall assume that Crusoe has at his disposal at time t an estimate, $\hat{S}_{t-AL}$, of the state of the system AL months earlier.

At this point it is desirable to introduce an assumption about the way Nature behaves in generating the harvests which Crusoe is attempting to forecast. This assumption, which will be carried in its essentials throughout the subsequent analysis, is that Nature produces "years" of monthly harvests according to a stationary stochastic process. A "year", naturally enough, consists of twelve monthly harvests. It is of no particular importance in which calendar month the year is taken to begin, and we therefore adopt the natural convention that month 1 is January, making month 12 December, month 13 January, etc. Our stationarity assumption amounts to saying that the probability distribution of January through December harvests corresponding to a random choice of calendar year is independent of the label on the year. Although this rules out the obviously realistic feature of a trend in the annual harvest this could easily be "tacked on" should the analysis require this complication.

What this assumption means is that any rule Crusoe might adopt for using crop information will lead in turn to a stationary stochastic process in twelve-month patterns of consumption. If we further (a) abstract from the particular starting point of Crusoe and (b) assume he is an expected money-value maximizer (indifferent to risk *per se*) we can evaluate alternative policies by computing the expected value, for any choice of r, of:

$$(3.3) \qquad \sum_{i=r}^{r+11} V(C_i)$$

with respect to the randomness due to nature and the randomness due to the imperfect forecasting instrument. For convenience we consider the expression for $N = 1$.

Let us consider now the nature of Crusoe's inventory determination rule. It seems clear that his inventory decision at any time must depend only upon how much grain he has held over from the previous month, how much is harvested in the current month, and the probability distribution of future harvests. Let us assume that Crusoe's decision in fact depends only upon the expected values of future harvests. Then the inventory he chooses to hold over from month t to $t+1$ can be written as a function R_t of inherited inventory, Q_{t-1} and forecast harvests, $\hat{G}_t, \hat{G}_{t+1}, \hat{G}_{t+2} \ldots$:

$$(3.4) \qquad Q_t = R_t(Q_{t-1}, \hat{G}_t, \hat{G}_{t+1}, \hat{G}_{t+2}, \ldots) \quad .$$

Furthermore, we know that forecast harvests are given by the appropriate elements of the vector $\hat{S}_{t-AL}$ for as far into the future as that vector extends. Beyond that date the forecasts are the expected values of nature's stationary distribution of monthly harvests.

The twelve-month harvest sequence which is the expected value of nature's distribution is sufficiently

important to deserve a name, and we have called it <u>the standard harvest pattern</u>, $(\bar{G}_1, \bar{G}_2, \ldots, \bar{G}_{12})$. It will be obvious that the last component S_t^{t+M+1} in the state of the system vector, S_t , is the standard harvest for the corresponding calendar month. We can therefore rewrite our rule (3.4) as

$$(3.5) \qquad Q_t = R_t^{AL}(Q_{t-1}, \hat{S}_{t-AL}^{t}, \ldots, \hat{S}_{t-AL}^{t+M-AL}) \quad .$$

The assumptions made thus far assure us that there will be at most 12 distinct rules R_t^{AL}; that is, the sequence of <u>functions</u>, $R_t^{AL}, R_{t+1}^{AL} \ldots,$ is periodic with period 12.

The form of rules (3.5) should not be taken to indicate that the inventory held at the end of a month does not depend upon the standard harvest pattern. Although those numbers do not appear among the listed arguments, this is simply because, for given t, the standard harvests always enter the calculations in exactly the same way. In fact, as we shall see when we come to the case of a market system, obtaining explicit expressions for rules (3.5) can be rather difficult in spite of the basically very simple model of harvest generation used.

Corresponding to nature's stochastic process producing the harvests there will be, via (3.5), a stochastic sequence of inventories. This stochastic process will also be characterized by a stationary distribution of twelve-month inventory

sequences. That is, there will be a stationary joint distribution of, say, the thirteen-month sequence of inventories stretching from December through December and the twelve harvests from January to December, independent of the calendar year. Using accounting identities (3.1) we can express $C_1,\ldots,C_{12}$ in terms of thirteen inventories $Q_o,\ldots,Q_{12}$ and twelve harvests $G_1,\ldots,G_{12}$. Substituting into expression (3.3) for the value of a twelve-month "piece" of a consumption process, we are in a position to compute the expected value of a nature's harvest process as translated into wheat consumption, given the information system and Crusoe's rules (3.5) for using information. The value of <u>improving</u> information is the amount by which expectation (3.3) is increased when the measurement errors are reduced, or the availability lag decreased.

To put these ideas into practice, it is necessary to make assumptions about function form. To illustrate, assume that V( ) is quadratic. It is a simple refinement to allow V( ) itself to depend upon the calendar month in which the consumption occurs. Accordingly, let $V_t(\ )$ be the valuation function for month t, it being understood that the sequence of functions V_t is periodic with period 12, and that all are quadratic. Assume further, for convenience only, that rules R_t are linear in their arguments.

Making the substitution of expressions (3.5) into objective function (3.3), and using the relationship (2.2) between S_t and $\hat{S}_t$, we have a quadratic expression in variable S_t, determined by nature, and Ψ_t, "determined" as the random errors of measurement associated with our forecasting system. If we hold all the variables other than Ψ constant, this substitution gives us a quadratic expression in the various errors of measurement. These errors are assumed to have the usual properties of independence from other variables in the system and of having an expected value of zero.

Under these various assumptions, the grand expectation of the objective function over nature's randomness and the randomness of the measurement system, can be expressed as a linear expression in first and second moments of nature's distribution alone, plus a linear expression in the second moments of the distribution of Ψ. As long as we are concentrating on the value of changing the moments of Ψ by changing the information system, the first expression can be ignored. When we come to considering the value of reducing the availability lag, AL, we shall need to inquire further into nature's distribution. This is best postponed until analytically more transparent evaluation of sample error reduction has been completed.

Rather than pursue Crusoe's problem further, having described the basic logic of the valuation of information, we turn now to a market model of inventory determination. In this model, society's decision rule analogous to R will be the result of profit seeking choices of inventory holders. We shall see that for "reasonable" specifications of the model and of the associated valuation function the expected value of the consumption stream is indeed a decreasing function of the variances of the sampling errors. Thus, if the market model is a reasonable approximation to the behavioral rules followed in practice, the direct benefits associated with a given level of sampling accuracy could be estimated if the behavioral model is so estimated.

4. The Social Value of Crop Forecasting Information in a Competetive Market System: Theory

In the previous section we sketched out a theory of the value of crop information in a one-man world. In this section we show how the same idea extends to a world in which grain is bought and sold in a marketplace and in which profit-maximizing inventory holders perform the determination of the amount of grain to be held from period to period. We continue to deal with a single commodity and to assume that the amount of grain harvested is entirely determined by Nature, so that the social problem remains the optimal choice of storage as before.

The principal ways in which markets enter the analysis are in the construction of the objective function and in the theory of the connection between inventory levels chosen and available information. Market prices are used in both problems. The markets which we introduce (besides the implicitly present capital market) are the spot and futures markets for grain.

The agents of our model are consumers of grain and inventory holders. In addition, grain speculators are, or may be, present. We shall assume that "consumers" are people who do not store significant quantities of grain, but rather use it up for current satisfaction or use it as an input to further production processes (e.g. in the form of cattle feed). We let

(4.1) $p_t(x)$ = the demand curve for grain = the price at which a quantity x of grain will be demanded for consumption.

Note that the demand curve is itself a function of time; we shall assume a different demand curve for each calendar month.

We shall assume that $p_t'(x)$ is negative -- the demand curve for grain for consumption (including use as feed) is negatively sloped. If C_t is the total amount of grain made available at time t, then the price, p_t which will rule in a competitive market-clearing situation at time t is

$$p_t = p_t(C_t) \quad . \tag{4.2}$$

The Objective Function

We shall take as the money equivalent to an amount x of consumption of wheat the area under demand curve (4.1) from zero to x:

$$V_t(x) = \int_0^x p_t(\xi)\,d\xi \quad . \tag{4.3}$$

As in the Crusoe Case, we can then represent the value of a twelve-month consumption sequence as

$$(4.4) \qquad \sum_{t=1}^{12} V_t(C_t) \quad .$$

As before, let Q_t represent the total amount of grain stored from period t to period $t+1$, and assume that a fraction $(1-\delta)$ of the stored grain is lost to insects, etc. Let G_t continue to represent the grain harvest in time period t. Then the consumption in t equals the grain harvested in that period plus "inheritance" from the previous period less inventories held over to period $t+1$:

$$(4.5) \qquad C_t = G_t + (1-\delta)Q_{t-1} - Q_t \quad .$$

Assuming no risk-aversion on the part of the social evaluator and subject to the usual qualifications about summing gains and losses to different individuals we can write as the objective of policy to maximize the expected value of annual consumption less storage costs:

$$(4.6) \qquad W = E\left[\sum_{t=1}^{12} (V_t(C_t) - TC(Q_t))\right]$$

where E is the expectation operator and, as before, $TC(Q)$ is the cost of storing Q_t units of grain for one period, with the C_t's conforming to (4.5). We think of the harvests G_t as specified by Nature, while the inventories

are determined by profit-maximizing inventory holders. By introducing improved information the choices of inventory holders are affected, with the resulting effect on welfare measured in dollars by the change in W .

5. The Relationship Between Inventories and Forecasts in a Competitive Model.

The Behavior of Inventory Holders

We assume there to be N inventory holders and let

(5.1) q_t^i = the amount stored by inventory holder i from t to $t+1$.

(5.2) $TC_i(q_t^i)$ = the period t dollar cost of holding q_t^i .

We shall assume that the same fraction δ of inventories is lost from period to period for all inventory holders.

Inventory holders attempt to make profits by buying cheap in one period and selling dear in the future, taking into account storage costs and deterioration of the grain in storage. We shall assume that inventory holders buy and sell in either the spot or future markets and that they are competitors, believing themselves able to buy and sell all they wish to at the quoted price. By assuming away transactions costs we can reduce the inventory holder's problem to a one-period one.

To develop this, first ignore the futures market and define

(5.3) $p^i_{t,t+j}$ = the spot price which, looking ahead from period t, is forecast by inventory holder i to prevail in period $t+j$.

Suppose that inventory holder i is currently holding q^i_t and is considering adding another ton to storage. He expects this will increase the amount he can sell next period by $(1-\delta)$ units, for which he anticipates he will receive $p^i_{t,t+1}(1-\delta)$ dollars. This is equivalent to $p^i_{t,t+1}(1-\delta)/(1+r)$ period t dollars in extra revenue, where r is the market rate of interest. This amount is to be compared with the sum of the extra purchase cost, p_t, and the extra storage cost, which we shall denote by $MC_i(q^i_t)$. If the difference is positive, there is an expected profit to be made from the procedure.

Hence assuming that the inventory holder is an expected profit maximizer we conclude that he will hold q^i_t only if

$$(5.4) \qquad p^i_{t,t+1} \cdot \frac{(1-\delta)}{(1+r)} - (p_t + MC_i(q^i_t)) \leqq 0$$

$$(\quad < 0 \quad \text{implies} \quad q^i_t = 0) \quad .$$

Condition (5.4) it will be noted, is the necessary condition

for maximization of profit from a one-period transaction, i.e., the maximization of $p^i_{t,t+1}\ q^i_t(1-\delta)/(1+r) - p_t\ q^i_t - TC_i(q^i_t)$, and it is not difficult to show that this maximization is necessary for the maximization of the expected present value of speculative profits from an entire sequence of inventory decisions.

Futures Markets Introduced

Suppose now that a futures market is available in which the inventory holder can, in effect, carry out the future sale or purchase in the present. Denote by $p_{t,t+1}$ the period $t+1$ price quoted on the futures market at period t. If we do not here concern ourselves overly with refinements of the theory of capital rationing, a condition of general equilibrium in a world of expected profit maximizers is

$$(5.5) \qquad p^i_{t,t+1} = p_{t,t+1} \qquad \text{for all } i \quad .$$

(More generally, condition (5.5) must hold for all traders in the futures market.) That is, at the margin all agents must have the same price expectation as that recorded in the futures market quotations. Hence our condition for individual profit maximization implies that

$$(5.6) \qquad p_{t,t+1} \cdot \frac{(1-\delta)}{(1+r)} - p_t - MC(q_t^i) \leq 0$$

$$(< 0 \Rightarrow q_t^i = 0).$$

Condition (5.6) simply characterizes the lack of opportunity for arbitrage by buying grain in one period and selling it forward at a price that more than covers the known storage plus waiting costs.

The important function of the futures market in this analysis is to coordinate expectations of different inventory holders. This will allow us to aggregate their choices.

Market Clearing

What determines the various prices, actual and expected? The actual current price is that determined by demand curve (4.1) to clear the market when the sum of new harvests plus old inventories less additions to inventories is offered for consumption. That is,

$$(5.7) \qquad p_t = p_t(G_t + (1-\delta) \sum_{i=1}^{N} q_{t-1}^i - \sum_{i=1}^{N} q_t^i) \quad .$$

The forecast prices could, of course, be anything, but we shall assume that they are derived from forecasts of quantities offered for sale in the future. Let

$$Q_t = \sum_{i=1}^{N} q_t^i \tag{5.8}$$

and let

(5.9) $Q^i_{t,t+k}$ = the forecast at time t by inventory holder i of Q_{t+k}, $k = 0, 1, 2, \ldots$.

Finally, let

(5.10) $G^i_{t,t+k}$ = the forecast at time t by inventory holder i of G_{t+k}, $k = 1, 2, \ldots$.

Then the market supply to consumers expected at period t by inventory holder i to prevail in period $t+1$ is given by

$$C^i_{t,t+1} = G^i_{t,t+1} + (1-\delta)\, Q^i_{t,t} - Q^i_{t,t+1} \, . \tag{5.11}$$

We shall assume that inventory holders behave as though endowed with knowledge of demand curve (4.1). This implies

$$p^i_{t,t+1} = p_{t+1}(C^i_{t,t+1}) \tag{5.12}$$

Thus to determine his current inventory, holder i must forecast next period's harvest and the aggregate inventory behavior this period <u>and</u> next period. Better crop prediction

affects his forecast of next period's harvest, but the inventory holder's use of this information depends upon the way in which he forecasts the behavior of other inventory holders. Thus to determine how information affects the flow of grain consumption in the market economy (equivalently in our model, the sequence of spot prices) we must construct a theory of the way in which the individual agent forecasts aggregate inventory holdings.

Inventory Forecasting in the One-Inventory-Holder Model

Things are simplified, notationally and otherwise, in the case in which there is only one inventory holder. Then we can drop the superscript and treat the aggregate inventory as identical to the individual agent's inventory. Our inventory holder's problem is in effect to predict his own behavior. An appealing assumption is that he will determine the principles guiding his current action and operate on the basis that he will use the same principle to determine his actions in the future.

Our agent has at time t a model for predicting the spot price of grain at any future date $t+k$, namely the appropriate version of expression (5.12) which we reproduce as:

$$(5.13)\quad P_{t,t+k} = P_{t+k}(G_{t,t+k} + (1-\delta)\cdot Q_{t,t+k-1} - Q_{t,t+k}) .$$

Furthermore, he knows that his inventory choice at, say, time t+k will be governed by the profit-maximization condition (5.6), which we reproduce as condition (5.14) on <u>forecast</u> Q_{t+k} .

$$(5.14) \quad p_{t,t+k+1} \left(\frac{1-\delta}{1-r} \right) - p_{t,t+k} - MC(Q_{t,t+k}) \leq 0$$

$$(< 0 \Rightarrow Q_{t,t+k} = 0) .$$

Using (5.13) twice we can express conditions (5.14) as a difference equation/inequality in forecast inventories:

$$(5.15) \quad \left(\frac{1-\delta}{1+r}\right) \cdot p_{t+k+1}(G_{t,t+k+1} + (1-\delta) \cdot Q_{t,t+k} - Q_{t,t+k+1})$$
$$- p_{t+k}(G_{t,t+k} + (1-\delta)\ Q_{t,t+k-1} - Q_{t,t+k}) + MC(Q_{t,t+k}) \leq 0$$
$$(< 0 \Rightarrow Q_{t,t+k} = 0) .$$

Hidden behind the forest of notation in (5.15) is a very simple relationship among $Q_{t,t+k-1}$, $Q_{t,t+k}$ and $Q_{t,t+k+1}$. In its equality form (5.15) is thus a second order difference equation. If we adopt the convention that

$$(5.16) \qquad Q_{t,t-1} \equiv Q_{t-1} ,$$

then condition (5.15) holds for k = 0, 1, 2, ...

Continuing to think of (5.15) in its equality form, we know that a second order difference equation has a solution unique up to the specification of two parameters. These are determined by two boundary conditions, frequently specified by given values of the first two terms in the sequence of values of the dependent variable. However, in this case we are given only one boundary condition, the inherited value of inventories, Q_{t-1}. The remaining condition must be provided by some sort of condition on $Q_{t,t+k}$ as k approaches infinity. The structure of the model alone at this point <u>does not</u> determine inventory choices. It is necessary to introduce further information or constraints on the formation of the inventory holder's expectations. We shall consider this problem now.

The discussion in the previous paragraph treated (5.15) as a difference equation. However, condition (5.15) may also hold as an inequality, in which case new features are introduced. In one respect these features are welcome, in that they help to provide the second boundary condition we need. In another respect they are unwelcome, since they introduce an inherent non-linearity into the relationship between inventories and crop forecasts, a complication for computation and for econometric work.

Recall that expression (5.15) is derived from condition (5.14) of profit maximization which says loosely that if you do not expect the price of grain to rise enough between now and next period to compensate for determination of stored stocks and cover storage and interest costs then you should sell off all your inventories today. Characteristically, ignoring inflation, the spot prices of agricultural crops go through a yearly cycle; in particular they drop when the main harvest is brought in. For those typical price patterns condition (5.14) holds as an <u>in</u>equality at least once per year. This has the plausible corollary that inventories are reduced essentially to zero at least once per year, just before the main harvest.

Of course, for crops which are sufficiently storable, this regular pattern may be broken for one or several cycles, during which stocks are never eliminated and <u>real price rises continually</u>. This might happen, for example, as a result of a succession of bad harvests. However, for the typical case the inventory holder's expectations, at least for the periods in the future beyond those for which he has current information, must be for a zero inventory level recurring at a regular cyclical interval. If we can develop an explicit model of when the first zero inventory level will be predicted to occur we shall have determined the solution of (5.15). Suppose,

for example, that we conclude $Q_{t,t+k^*} = 0$, and for $k < k^*$, $Q_{t,t+k} > 0$. Then between t and $t+k^*-1$ expression (5.15) holds as an equality. Taken together these conditions will determine the values of $Q_{t,t}, Q_{t,t+1}, \ldots, Q_{t,t+k-1}$. Of these our interest is really only in $Q_{t,t}$, the current inventory decision.

Inventories Non-Linear in Forecasts

An unfortunate feature of this model of the determination of inventories is that for the simple linear version of (5.15) in which we would obtain inventories linearly dependent upon crop forecasts if (5.15) held as an equality we now obtain a non-linear relationship. This is easily illustrated. Let us suppose that the available evidence predicts a bumper harvest next period. And let us suppose that this prediction places beyond a shadow of a doubt the conclusion that the real price of grain will decline between this period and next. My optimal policy as an inventory holder, then, is to sell off any stocks I may have today. Now consider the value to me of improved accuracy in the prediction of next period's crop. Since I am already certain that the price will decline, my action will not in the least be affected by pinpointing exactly how much the price

will decline. My response -- eliminating my inventories -- is non-linear in crop forecasts.

Because the response of inventories is non-linear in this way, so is the value of information about crops. Often even rough forecasts may make it clear that the price will decline next period. Increased forecast accuracy is only valuable for the cases in which a difference in the forecast leads to a different decision, which is to say in which the current inventory is non-zero.

This sort of non-linearity generalizes. Let us suppose that, on the basis of current information I now, in March, say, expect inventories to be driven to zero at the end of June as the price falls with a large incoming harvest in July. Quite plausibly, even rather large changes in my expectation for the July, August or September harvests would not affect my expectation that inventories will be zero at the end of June. Only by changing the month in which inventories are expected first to fall to zero, can changes in forecast harvests beyond that date affect my current decision. On the other hand, changes in any of the monthly harvests forecast to occur <u>before</u> the end of the month at which inventories go to zero <u>do</u> affect the current inventory. Accuracy in these forecasts is correspondingly valuable.

There remains the possibility that forecast error could lead to the wrong month being predicted as the date on

which inventories are first zero. Let us suppose, for example, that I am making my inventory decision in March and I know the harvests with perfect accuracy into the distant future. Suppose further that in view of these forseen harvests I expect to hold positive inventories beyond the current harvest year, with inventories expected to be zero at the end of May, a year and two months hence. Now consider how my decision is affected by a changed forecast of the harvest expected in the coming August. As that anticipated harvest increases, the need to hold current wheat for consumption beyond the coming August decreases, reflected in my market predictions by reduction in the expected price beyond that month. There will be a critical level of the anticipated August crop (given the levels of the remaining months' harvests) below which I shall plan on having my inventories run down to zero at the end of the approaching May, two months hence, and above which I shall plan on having my inventories run down to zero twelve months later. It is thus possible that errors of measurement could lead us to guage incorrectly the earliest zero-inventory date.

Review of the Reasoning

Let us recapitulate. The difference equation/inequality system (5.15) is an expression of the no-profitable-arbitrage-possible characteristic of speculative market equilibrium coupled with an assumption that inventory holders behave as though they know the demand and marginal cost structure of the market. This system constrains at each date the current inventory as well as a sequence of anticipated inventories, given an inherited inventory, and a sequence of anticipated monthly harvests, including the current one. Suppose we know at time t the earliest time $t+k^*$ at which the inequality of system (5.15) holds. (Incidentally, any other method of determining a future inventory would do, provided we could be sure the intervening constraints hold as equalities.) With this information the sequence of inventories $Q_{t,t}$ (the current decision), $Q_{t,t+1},\ldots, Q_{t,t+k^*-1}$ (the forecast future decisions), is completely determined, since the k^* conditions (5.15) corresponding to these inventories hold as equalities. We are not really interested in the forecast future inventories, but these must be determined simultaneously with current inventories. The subset of conditions (5.15) just singled out determines $Q_{t,t}$ to $Q_{t,t+k^*-1}$ implicitly as functions of Q_{t-1} and $G_{t,t}$ through $G_{t,t+k^*}$. With a sufficiently simple structure, this system can be solved explicitly, giving

us an expression for $Q_{t,t}$ in terms of inherited inventories, present and forecast future harvests. Knowing the distribution of errors of forecast due to measurement, we can translate these into a distribution of current inventory decisions, given k*.

If, instead of a single number, k* , we are given a distribution of numbers, we can, clearly, repeat the procedure just described for each value of k* , and compute the resulting distribution of current inventory. All that is left out is the possibility that forecast error causes an incorrect choice of k^*. As we shall see when we turn to the determination of k* , it depends on Q_{t-1} and the sequence of forecast present and future harvests. This means that we could, in principle, compute a distribution for $Q_{t,t}$ from a knowledge of the standard harvest pattern, actual inherited inventories, nature's distribution of shocks to the standard harvest patterns, and the properties of the errors in the forecasting system. Even under simple assumptions as to functional form, however, the calculations would have to be numerical and would be exceedingly complex. The method we have chosen to simplify this procedure has as a weak point the necessity of neglecting the interaction between measurement error and the determination of k* . As far as we can tell the bias thus introduced is small and of undeterminate direction.

Once we have established for each month t a distribution of the associated parameter $k^*(t)$ our system (20) determines a distribution of $Q_{t,t}$. This is the "rule" by which the one-inventory holder market system relates forecast harvests to current inventories, the relationship required in order to evaluate the worth of improvements in forecast accuracy. Before we turn to the derivation of the distribution of $k^*(t)$, let us turn briefly to the question of what adjustment needs to be made to re-introduce many inventory holders.

Inventory Forecasting in the Many-Inventory-Holder Model

To generalize the preceding analysis to a world of many inventory holders, we again appeal to the ability of our model agents to solve implicitly rather difficult mathematical problems. In this case we rely on their being able to convert a quoted sequence of spot and future prices for grain, via a knowledge of the demand function for grain and the supply function of storage (the economy's marginal cost of storage function), and a knowledge of forecast current and future harvests, into the consistent sequence of current and forecast aggregate inventory levels. This is the second point at which we have used both an implausible knowledge of the structure of the economy and an implausible capacity for cal-

culations, in developing our model. While it would be desirable to have a more "realistic" theory in this regard, however, it is not clear that the obvious sorts of rule of thumb models of behavior (trend extrapolation, etc.) are superior, and they would, we think, be, an impediment to clarity in the picture we are to draw.

Our inventory holder speculators are operating in this model with an estimate of the factual state of affairs which is consistent with the information from the forecasting system and the known prices quoted on the various markets. This consistency is desirable as proof against results which follow from ad hoc assumptions. None the less, further attention to the aggregation problem would be desirable.

Closing the Dynamic System[1]

We conclude this section by discussing the way in which the missing second boundary condition for the system (5.15) is obtained. We assume that the inventory holders use the announced forecast harvests for as far into the future as these can be calculated from known information. Recalling the discussion in Section 2, we regard inventory holders as replacing $G_{t,t+i}$ in (5.15) by $\hat{S}^{t+i}_{t-AL}$ for values of i up to M. Beyond that point in the future inventory holders are assumed to adopt as forecasts simply the *a priori*

[1] Readers may wish to omit the rather technical discussion in this and the next subsections and go directly to Section 6.

harvest pattern which we have called the standard harvest pattern. This pattern, it will be recalled, is periodic, with a period of twelve months.

Consider first the case in which the forecast harvest sequence is itself the standard harvest sequence. It can be shown that corresponding to any given periodic harvest sequence there exists a sequence of inventories, which is itself periodic with period twelve and which satisfies conditions (5.15). Furthermore we know that the inventory sequence has at least one zero element, to which corresponds a strict inequality in (5.15). Furthermore, while we have not yet attempted to prove this, it seems likely (since storage is costly, in effect a dampening force) that given any non-negative inherited inventory Q_{t-1}, there is a solution to system (5.15) which is ultimately purely periodic. The system tends toward a steady-state inventory path. That is, given any Q_{t-1}, if the harvests $G_{t,t+i}$ describe a periodic path, for J large enough there is a solution to (5.15) such that $Q_{t,t+j}$ follows the steady state path for $j \geq J$.

Intuitively speaking, if we look far enough into the future, assuming no trend in harvests, we must bet that at the end of May inventories will be zero if, on average, crop flow begins to build up sharply at that time. This provides us with a date, $t+k^{**}$, such that $Q_{t,t+k^{**}}$ must be zero.

This is obviously a step forward, but it does not provide us immediately with the ability to solve system (5.15) for the values of $Q_{t,t}$ to $Q_{t,t+k^{**}-1}$, of which $Q_{t,t}$ is our true objective. What we require is the smallest integer, which we have called $k^*(t)$, such that $Q_{t,t+k^*(t)} = 0$. Given k^*, the problem reduces to one of solving a system of equations.

Finding a Market Solution

Suppose we had a solution to system (5.15) augmented by the condition $Q_{t,t+k^{**}} = 0$, where by "solution" we mean a sequence $Q_{t,t}, \ldots, Q_{t,t+k^{**}-1}$ such that for each element either the corresponding constraint is binding (and hence satisfied as an equality) or not binding (in which case the corresponding $Q_{t,t+j}$ is zero). All we would need to do to determine the inventory carry-over horizon, $k^*(t)$, would be to look for the first non-binding constraint. For example, if the very first condition is satisfied as an inequality, we would say $k^*(t) = 0$: the number of months remaining until inventories are sold off to zero is zero.

Once we have established k^{**}, then, all we need to do is find a feasible solution to the inequality system consisting of the k^{**} conditions on $Q_{t,t}$ through $Q_{t,t+k^{**}-1}$ from system (5.15). Although in principle a simple matter, finding such soultions is not a standard computational procedure. The problem can, fortunately, be converted to one

for which well developed computational routines exist by recognizing that a solution to our k^{**} inequalities (with the prescribed non-negativity and complementary slackness properties) corresponds to an optimum of a non-linear programming problem. We simply take as the objective function of this artificial problem the sum of the products of each $Q_{t,t+j}$ with its corresponding constraint function. We then attempt to maximize this sum of products subject to (a) the non-negativity of inventories, $Q_{t,t+j}$, and (b) the satisfaction of our k^{**} inequalities from (5.15). If there is a solution to our original problem this derived problem will also have a solution and will yield an objective function value of zero. This is so because a solution to our original problem has non-negative inventories, satisfies (5.15), and has a zero value of the constraint corresponding to any positive inventory. The sum of products of inventories and their constraints is thus zero at a solution to the original problem.

That this is the maximum value of the objective function to our derived programming problem follows from the fact that is is feasible and that the value of the objective function for any feasible vector of inventories is non-positive (inventories being non-negative and constraint functions being non-positive). Since zero is as large as the sum of products can be the feasible solution to our original problem is an optimum of the derived one.

The logic works the other way. If we can find a solution to the derived problem which has the value of its objective function equal to zero, the inventories form a feasible solution to our original problem. The lowest numbered constraint satisfied as a strict equality corresponds to $Q_{t,t+k*}$. In short, we shall have computed $k*$.

If the derived problem is not feasible or has an objective function value less than zero at its optimum then the original problem does not have a solution. This provides us with a convenient check on the "reasonableness" of empirical specification of system (20), to which we now turn.

6. Empirical Implementation in a Linear Model of the U.S. Wheat Market

In the preceding section we have developed a method for evaluating measurement improvements in forecasting crops. The model includes demand functions for grain to consume or use as an input and a cost function of grain storage. To implement the analysis we require empirically estimated versions of these, together with observations or assumptions about the discount rate, r, and the rate of deterioration, δ. Finally, in order to establish the month-by-month distribution of k^*, the inventory carry-over horizon, we require a specification of the way in which Nature is assumed to generate grain harvests. All of these empirical data have been assembled for the case of wheat crop forecasting in the United States, using linear specifications for the demand and marginal cost functions. The details of these estimations and of some of the derivations have been placed in appendices for easy reference. In this section we shall attempt to describe in a compressed fashion how all the pieces fit together to produce an estimate of the value of reducing measurement error.

It bears repeating at this point that we have viewed the calculation of a single number to represent the value of better wheat information as secondary to the development of a sound, empirically implementable method for performing such calculations. Such an emphasis will justify the length to

which we have gone here to explain procedures and reasoning. (This is not to say the empirical results are merely "illustrative.")

Assume, then, that demand and marginal cost functions are given by

$$\text{(6.1)} \qquad \begin{aligned} p_t(c) &= a_t - bC \\ MC(Q) &= d + eQ\,, \end{aligned}$$

where a_t is periodic with period 12 ($a_{t+12} = a_t$ for all t). (These expressions could, of course, be linear approximations to relationships which are actually non-linear.) Our system (5.15) becomes then a linked series of linear inequalities in which forecasts enter as constant terms, determining the intercepts. These inequalities can be written as (for $k = 0,1,\ldots$),

$$\text{(6.2)} \qquad [A_1, A_o, A_{-1}] \begin{bmatrix} Q_{t,t+k+1} \\ Q_{t,t+k} \\ Q_{t,t+k-1} \end{bmatrix} \leqq D_1 G_{t,t+k+1} + D_o G_{t,t+k} + F_{t+k}\,,$$

where

$$A_1 \equiv b \frac{(1-\delta)}{(1+r)}$$

$$A_o \equiv - b \left(\frac{(1-\delta)}{(1+r)} + 1\right) - e$$

$$A_{-1} \equiv b (1-\delta)$$

$$D_1 \equiv b \left(\frac{(1-\delta)}{(1+r)}\right.$$

$$D_o \equiv - b$$

$$F_{t+k} \equiv a_{t+k} - \frac{(1-\delta)}{(1+r)} a_{t+k+1} + d ,$$

and where

$$(<) \implies Q_{t,t+k} = 0 .$$

Note that, because the coefficients a_i are periodic with period 12, so are coefficients F_{t+k}.

The parameters a_t and b were estimated for the total "domestic disappearance" of wheat in the United States. (See Appendix A for details.) The marginal cost of storage function was estimated from time series data on the spread between spot and futures prices and total stocks on hand in the United

States. (See Appendix B.) Efforts to estimate δ empirically were unsuccessful. Persons knowledgeable in the wheat market regard δ as effectively zero, and this was the value used in our calculations. The discount rate, r, used in the estimation was the rate of interest for prime commercial paper. Over the sample period for the demand function estimates, 1955-1971, it averaged roughly .005 per month. This was therefore used in the evaluation procedures. These data and the derived values for the parameters of system (6.2) are presented in Table 6.1.

If we are now given a value for $k^*(t)$ we can, using (6.2) express $Q_{t,t}$ as a linear combination of the inherited inventory, $Q_{t,t-1}$ $(= Q_{t-1})$ and "forecasts," $G_{t,t}, G_{t,t+1}, \ldots, G_{t,t+k^*}$. The term "forecasts" is in quotation marks because, in general, only the first few months of harvests will be forecast on the basis of actual data (how many depends upon AL, the availability lag). The remainder will consist of the appropriate sequence of elements from the standard harvest pattern, $\bar{G}_1, \ldots, \bar{G}_{12}$.

The social objective function in the linear model becomes a quadratic in consumption and inventory levels:

$$(6.3) \qquad \sum_{i=1}^{12} \left[\int_0^{C_i} p_i(\xi) d\xi - \int_0^{Q_i} MC(\xi) d\xi \right]$$

$$= \sum_{i=1}^{12} \left(a_i C_i - \frac{b}{2} C_i^2 - dQ_i - \frac{e}{2} Q_i^2\right)$$

TABLE 6.1: Parameters Used in Evaluation and Monte Carlo Calculations

$$p_t = a_t - b\,C_t \qquad (p_t \text{ in 1958 cents, } C_t \text{ in millions of bushels per month})$$

a_1	= 362.2	a_7	= 384.0
a_2	= 374.1	a_8	= 421.6
a_3	= 352.3	a_9	= 393.9
a_4	= 330.6	a_{10}	= 366.2
a_5	= 308.8	a_{11}	= 338.5
a_6	= 346.4	a_{12}	= 350.3

$$b = 4.3851$$

$$MC = d + e\,Q_t \qquad (MC \text{ in 1958 cents, } Q_t \text{ in millions of bushels})$$

$$d = -0.0207$$
$$e = 0.0003349$$

A_1	= 4.3633	F_1	= -10.06	F_7	= -35.52
A_o	= -8.7488	F_2	= 23.53	F_8	= 29.64
A_{-1}	= 4.3851	F_3	= 23.32	F_9	= 29.50
D_1	= 4.3633	F_4	= 23.32	F_{10}	= 29.36
D_o	= -4.3851	F_5	= -35.90	F_{11}	= -10.08
		F_6	= -35.71	F_{12}	= -10.12

We use identities (4.5) to express the social objective function as a quadratic in inventories, Q_o to Q_{12}, and actual harvests.

Each inventory in turn is expressable via the system (5.15) (recall the assumed sequence of values of $k^*(t)$) as a linear function of forecasts and once-lagged inventory. The forecasts are either elements of the standard harvest pattern or <u>estimates</u>, $\hat{S}^{t+k}_{t-AL}$, of the ideal forecasts, S^{t+k}_{t-AL}, as described in Section 3. Recall that $\hat{S}_{t-AL}$ deviates from S_{t-AL} by a vector of measurement errors, ψ_{t-AL}. Our linear expressions for inventories in terms of forecasts and lagged inventories can thus be replaced by linear expressions in lagged inventories, ideal forecasts, and measurement errors.

Since we assume that true ideal forecasts and measurement errors have distributions which are periodic with period 12 (e.g., loosely speaking, the error of observation of June's harvest always has the same variance, ditto for May, etc.), inventories will also have distributions which are periodic. For example, the expected value of $(Q_o)^2$ will be the same as that of $(Q_{12})^2$. Making the substitution of Q_{12} for Q_o, we can express the twelve inventories, Q_1 to Q_{12} as linear functions of ideal forecasts and measurement errors.

It will be recalled that in Section 3 we introduced the assumption that measurement errors have expected value zero and are distributed independently from each other and all other variables of the system. A consequence is that we can now express the variance of inventories as a linear function of variances and covariances of ideal harvest forecasts plus a linear function of variances of the errors of measurement. When we make the further assumption that the distribution of the error of measurement depends only upon the month of the harvest being measured (and not on the month in which the measurement is taking place) we reduce the number of measurement random variables to twelve (and several of these will be identically zero). We denote the variance of the error of measurement of month i's ideal harvest as ER(i). Table 6.2 presents as an illustration the coefficients of each of the twelve monthly error variances (across the rows) in the linear expression for January and June inventory variances for the case of the structural parameters from Table 6.1. Two cases are shown. The first assumes the k* sequence: (k*(1), k*(2), ..., k*(12)) = (4, 3, 2, 1, 0, 11, 10, 9, 8, 7, 6, 5), and an availability lag of zero. This k* sequence is the simplest one, in which it is always anticipated that inventories will be zero at the end of the next following May. The second assumes the k* sequence (40, 39, 38, 37, 36, 35, 34, 33, 32, 31, 30, 29), and availability lag zero.

Table 6.2: Coefficients of the Twelve Monthly Measurement Errors on the January and June Inventory Variances

Case 1. AL = 0 k* = 4,3,2,1,0,11,10,9,8,7,6,5

Case 2. AL = 0 k* = 40,39,38,37,36,35,34,33,32,43,42,41

Case 3. AL = 1 k* = 4,3,2,1,0,11,10,9,8,7,6,5

Case 4. AL = 1 k* = 40,39,38,37,36,35,34,33,32,43,42,41

Coefficient of ER Error of Measurement of Harvest in Month i

	Inventory Variance	ER(1)	ER(2)	ER(3)	ER(4)	ER(5)	ER(6)	ER(7)	ER(8)	ER(9)	ER(10)	ER(11)	ER(12)
Case 1.	1	.68146	.07764	.07671	.07539	.07345	.11550	.13736	.16661	.20622	.26179	.34325	.46964
	6	.00656	.00655	.0	.0	.0	.8438	.00661	.00660	.00659	.00658	.00657	.00656
Case 2.	1	2.10690	1.20809	1.26691	1.33028	1.39866	1.47260	1.55355	1.64156	1.73749	1.84234	1.92462	2.01262
	6	1.63518	1.71305	1.79676	1.88765	1.98577	2.09190	1.20760	1.27583	1.35021	1.43151	1.49486	1.56261
Case 3.	1	.68146	.07675	.07542	.07347	.07044	.11515	.13736	.16661	.20622	.26179	.34325	.46964
	6	.00656	.0	.0	.0	.0	.84384	.00661	.00660	.00659	.00658	.00657	.00650
Case 4.	1	2.10598	1.20706	1.26576	1.32898	1.39719	1.47175	1.55262	1.64054	1.73637	1.84167	1.92387	2.01179
	6	1.63387	1.71159	1.79586	1.88665	1.98463	2.09124	1.20688	1.27504	1.34934	1.43055	1.49379	1.56143

We know that the shorter is the availability lag, the greater is the length of the "future" we can see, and hence the more measurement errors have a chance to affect current inventories. For example, the coefficients of the twelve error variances in the expressions for the January and July inventory for the two k^* sequences with $AL = 1$ instead of $AL = 0$ are shown as in Case 3 and Case 4 of Table 6.2. These can be compared with Case 1 and Case 2, respectively.

We observed that the social objective function could be written as a quadratic expression in twelve random inventories and ideal forecasts. The expected value of the social objective function will then be linear in the means, variances and covariances of the variables. We can think of that expectation as the sum of an expectation of the value of the objective function, given perfect information (no measurement error) minus a term representing the loss in value attributable to measurement error. It turns out that the latter can be expressed as a linear combination of the variances in inventories due to measurement errors. Specifically, the loss due to measurement error is given by

$$(6.4) \qquad (b + \frac{e}{2}) \sum_{i=1}^{12} \sigma^2_{Q_i} + b \sum_{i=1}^{12} \pi_{i+1} \sigma^2_{Q_i} \quad ,$$

where $-b$ and e are the slopes of demand and marginal storage cost functions, respectively, π_i is a coefficient capturing the covariance of successive inventories, and the variances in the expression are conditional upon everything except the errors of measurement, i.e., they are due to errors of measurement.* Since the variances in (6.4) can themselves be expressed as linear combinations of the variances of the measurement errors we can, finally, express the loss in expected value of the social objective function due to measurement error, as a linear combination of measurement errors also. As before, these coefficients will depend upon the parameters of the system as in Table 6.1. and on the assumed sequence of inventory carry-over horizons, $k^*(t)$, as well as on the availability lag. Table 6.3 illustrates, for the same series of cases of $k^*(t)$ and availability lag defined in Table 6.2, the coefficients of the twelve measurement errors in the expected loss of social value expression. (Cases 5 and 6 in Table 6.2 will be discussed shortly.)

The next step in the process is to obtain a distribution of the sequences of inventory carry over horizons, $k^*(1),\ldots,k^*(12)$. This was obtained in Monte Carlo simulations of the operations of the wheat market; other grain markets can also be simulated. Carrying out the Monte Carlo simulation, a major undertaking, required, in addition to the parameters

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* For details, see Appendix C.

Table 6.3: Coefficients of Twelve Monthly Measurement Errors in Expected Loss of Social Value Under Various Assumptions

Case 1. AL = 0, k* = 4,3,2,1,0,11,10,9,8,7,6,5
2. AL = 0, k* = 40,39,38,37,36,35,34,33,32,43,42,41
3. AL = 1, k* = 4,3,2,1,0,11,10,9,8,7,6,5
4. AL = 1, k* = 40,39,38,37,36,35,34,33,32,43,42,41
5. AL = 0, Monte Carlo average
6. AL = 1, Monte Carlo average

Coefficient of ER(i), Variance of Error of Measurement of Harvest in Month i

	ER(1)	ER(2)	ER(3)	ER(4)	ER(5)	ER(6)	ER(7)	ER(8)	ER(9)	ER(10)	ER(11)	ER(12)
Case 1.	-2.0095	-2.0067	-1.9873	-1.9629	-1.9286	-2.0190	-2.0182	-2.0173	-2.0162	-2.0149	-2.0134	-2.0117
Case 2.	-2.1330	-2.1303	-2.1274	-2.1243	-2.1210	-2.1174	-2.1145	-2.1114	-2.1081	-2.1402	-2.1380	-2.1356
Case 3.	-2.0095	-1.9910	-1.9684	-1.9396	-1.8994	-2.0190	-2.0182	-2.0173	-2.0162	-2.0149	-2.0134	-2.0117
Case 4.	-2.1316	-2.1288	-2.1258	-2.1226	-2.1192	-2.1164	-2.1135	-2.1103	-2.1069	-2.1391	-2.1367	-2.1342
Case 5.	-2.0581	-2.0629	-2.1139	-2.0058	-2.1126	-2.1375	-2.1410	-2.1337	-2.1239	-2.1312	-2.1241	-2.1030
Case 6.	-2.0534	-2.0556	-2.1057	-1.9957	-2.0998	-2.1363	-2.1394	-2.1317	-2.1215	-2.1284	-2.1201	-2.0995

of demand and marginal cost functions already described, specification of the random process by which harvests are generated. Key elements of this process are the standard harvest pattern, and the parameters of a set of shocks by which Nature is assumed to convert the standard harvests into actual harvests in a sequence of steps.

In producing the Monte Carlo simulation it was necessary to deal with one refinement which is relevant to the subject of this section as well. Thus far we have been assuming that the only uses of grain are for consumption or addition to inventory. For a closed economy, or, alternatively, for a model of the world grain market this dichotomy would be sufficient. However, as our application will be to domestic U.S. consumption and inventory behavior, we must introduce a third use of grain, "exports." We recognized that a fully satisfactory incorporation of the foreign trade in grain to our theoretical and, more especially, to our empirical analysis would introduce a very substantial increase in its complexity. We therefore elected to use a naive model of export determination, assuming

$$EX_i = f + gH_i \quad , \tag{6.5}$$

where EX_i is the quantity of wheat exported in month i , and H_i is the actual amount harvested in month i .

The data on exports make it clear that there is little if any tendency for them to follow the seasonal pattern of harvests. In fact, the average exports for the 1965-72 period for each of the four quarters were virtually identical. The naive model (6.5) is thus obviously not a good one if taken literally as a monthly model. However, it is a reasonable one on an annual basis, saying simply that some portion of the variation in actual harvests, up or down, will be cushioned in its effects on domestic consumption by adjustment in exports. For purposes of the Monte Carlo study, the inaccuracy of the month-by-month pattern of exports generated by model (6.5) was deemed unimportant, while for the later use we shall make of that model in this section principle interest attaches to the coefficient of actual harvests, H_i, which will be the same for monthly and annual models. The details on estimation of the parameters of (6.5) lead to the following results:

$$(6.6) \qquad \hat{f} = 8.6, \qquad \hat{g} = 0.425$$

Table 6.4 summarizes the harvest pattern used as the basis for the Monte Carlo study; included as well are "steady state" export and effective harvest patterns for subsequent use in the analysis.

6.11

Table 6.4 Standard Harvest Pattern Used in the Monte Carlo Study to Determine Distribution of Inventory Carry Over Horizon, Along with Steady State Exports and Effective Harvests

	(Millions of Bushels)		
	(1) Harvests ($\overline{H}_i$)	(2) Exports ($\overline{EX}_i$)	(3) Net Effective Harvests (=(1)-(2)) ($\overline{G}_i$)
January	0.0	58.2	-58.2
February	0.0	58.2	-58.2
March	0.0	58.2	-58.2
April	0.0	58.2	-58.2
May	12.3	58.2	-45.9
June	453.0	58.2	394.8
July	492.7	58.2	434.5
August	374.9	58.2	316.7
September	76.9	58.2	18.7
October	6.4	58.2	-51.8
November	0.0	58.2	-58.2
December	0.0	58.2	-58.2

Note: Total Harvests and Exports Represent Averages for the Years 1965-1972.

In the Monte Carlo study the harvests in Column (1) of Table 6.4 were subjected to shocks before exports were determined by (6.5) and subtracted in any month to yield a net effective harvest for domestic purposes. The distribution of these shocks was estimated from data on annual harvest variance.

Using these parameters a fifty year "history" of the system was generated with the primary objective to obtain a distribution of the sequences of k*. The results could be discussed at great length.* These are interesting on their own, but we simply note here how very much the horizon shifts over time, a result in part of the very low cost of storing wheat. According to this model, holding periods of over three years are not unexpected. To each k* sequence corresponds a set of coefficients such as in Table 6.3 By calculating all of these coefficient sets and averaging them together in the proportions in which the k* sequences occurred in fifty year simulated history, we obtained the expected value of twelve coefficients of monthly measurement error in the calculation of social loss. These are listed, for AL = 0 AL = 1 as Cases 5 and 6, respectively, in Table 6.3.

We are now at the point at which all we need to estimate the loss to the economy due to forecast measurement error

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* Actually two twenty-eight year histories were run. The first three years of each were discarded to eliminate any bias introduced by the start-up position.

is a set of twelve measurement error variances. These statistics are unfortunately elusive. Part of the difficulty results from our use of the concept of an ideal forecast, e.g. S_{23}^{26}, which is not directly observable. Thus we cannot simply look at a series of estimates, $\hat{S}_i^j$ and compare them with the after-the-fact known values S_i^j in order to estimate the error variance. In order to construct observations of true values of S_i^j we should have to know the precise components of the forecasting formula used (in this case by the USDA) and to have available a series of before- and after-the-fact values for these components. From after-the-fact values of the components one could calculate an ideal forecast.

A key example of such a component is planted acreage. This statistic is used in the formula for constructing forecasts, and it is especially with respect to estimating this statistic that satellite technology offers great advantages. For illustrative purposes let us suppose this is all the information that is required to make a forecast. The acreage of a crop planted at a specified time is recorded in successive months as it varies due to changing farmer decisions, weather vagaries, etc. At each point a forecast of the harvest from this planting is made by multiplying the acreage by some biologically determined constant. In this illustrative case, any error in measuring

the acreage translates into an equal percentage deviation between actual and ideal forecasts of the harvest from that planting.

The example is apt in illustrating a difficulty in estimating measurement errors even of the component, in this case acreage. For there is no "true" acreage figure ever discovered. We cannot simply compare a measured and actual series. Rather measurement errors have to be guessed at by applying a statistical theoretical model to the sampling procedure.

Errors in estimating acreage will be only one source of deviation between actual and ideal forecasts. Information can be obtained as a crop matures which enable the yield per acre to be forecast. If this information is subject to error it will also cause a deviation between actual and ideal forecast. (Keep in mind that even the ideal forecast is subject to Nature's variability, the unpredictable in the future.) Roughly speaking, if the errors of measurement of yield and acreage are independent the variance of the deviation between ideal and actual forecast will be the sum of the variances of the two errors of component measurement.

Lacking adequate measurements of the errors of measurement of ideal forecasts at this point we must present a parametric summary of results. The coefficients summarized in Cases 5 and 6 of Table 6.3 in effect already present a parametric set of answers, but the number of parameters is unwieldy. That formula gives us the value of the loss due to

6.15

measurement error as a function of the twelve monthly error variances, ER(1),...,ER(12) . This may be further simplified if we assume further that the errors in any forecast tend to be proportional to the true value.

Recall that the actual forecast at time i of the harvest at time j , $\hat{S}_i^j$ differs from the ideal forecast, S_i^j , by the measurement error ψ_i^j . We assume that the standard deviation of ψ_i^j is proportional to $\overline{H}_j$, where $\overline{H}_j$ is the standard actual harvest for month j . Specifically, assume that

$$\text{(6.7)} \qquad ER(j) = \text{variance } (\psi_i^j) = \left(\frac{\alpha}{1.96}\overline{H}_j\right)^2 .$$

With this assumption we are saying roughly that the estimated forecast will differ from the ideal forecast for that month by less than 100α percent 95% of the time.

It is apparent from (6.7) that the loss to the economy due to measurement error will be simply proportional to α^2 . The estimated expected coefficient of α^2 is 3306.7 for the case of AL = 1 and 3309.0 for the case of AL = 0, where loss is measured in millions of dollars per year. The lowest curve in Figure 6.1 graphs the relationship for AL = 0. The equation is LOSS = $3309.1\alpha^2$.

These results indicate that starting from a measurement error that is within 10% about 95% of the time and moving

ing to zero measurement error would be worth 33,091,000 1958 dollars per year in perpetuity if the wheat system were basically stationary at the level of the late 1960's.

Adjusting to 4th quarter 1973 price level makes the relationship LOSS = $5294.4\alpha^2$, in millions of dollars, graphed as the middle curve in Figure 6.1.

It will be useful to make some adjustment for the fact that the actual system for which the value of information is being sought is a growing one. While there is some looseness in making a simple adjustment for this since the distribution of k^* was obtained in a (stochastically) stationary model and since population and time variables enter explicitly to the estimated demand functions, it should be roughly the case that in an economy in which the population is growing a 2% per year, the expected losses due to measurement error, instead of being a constant annuity, will be an annuity growing at 2% per year. To convert this growing stream of losses into an equivalent constant annuity, we require an assumed discount rate. Without wishing to become involved in the controversy over the appropriate social discount rate, but at the same time wishing to reduce the number of free parameters to be carried along in describing our results, we have assumed a discount rate of 6% (in real terms). This implies that the losses thus far should be increased by 50%.* The

* For those wishing to substitute their own assumptions about population growth and discount rate, the multiplicative factor is $r/(r-\rho)$, where r is the discount rate and ρ the population growth rate.

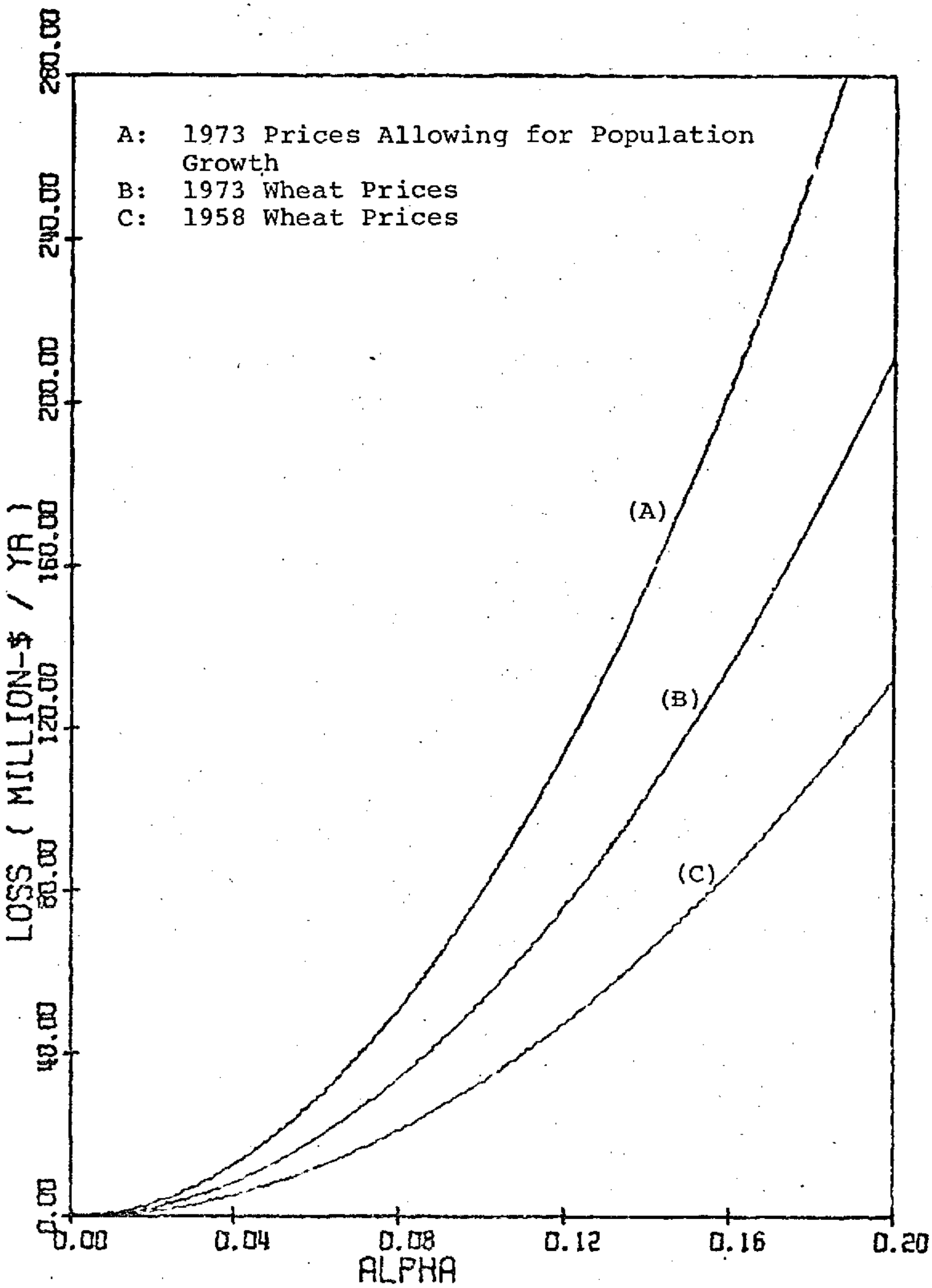

Figure 6.1: Annual Loss When Measurement Percentage Standard Deviation is Alpha/2, Assuming AL = 0

resulting relationship between measurement parameter, α, and expected loss due to measurement error, is shown as the uppermost curve in Figure 6.1.

One further adjustment is desirable, to account for the tendency for variation in actual harvests to be compensated for by offsetting changes in exports. The loss estimates thus far have been based on a factor of proportionality between the average actual harvests by month and the 95% confidence interval on measurements. These measurement errors will not translate into equivalent errors in the ideal forecasts of effective harvest, actual harvests less exports. According to our estimated naive model a unit change in actual harvest will tend cause on average a change of .575 units of effective harvest. To adjust for this we must multiply the expected losses, which are linear functions of the measurement error variances of effective harvests, by $(.575)^2 = .331$. The resulting relationship between loss and the factor referring to errors of measuring actual harvest is given by LOSS $= 2628.7\alpha^2$, and graphed in Figure 6.2.

It is obvious that the worth of improved information is highly sensitive to the value of α, and it would be most desirable to have accurate information about both its current value and the sorts of improvement obtainable through satellite technology. We must strongly emphasize that adequate

C-2

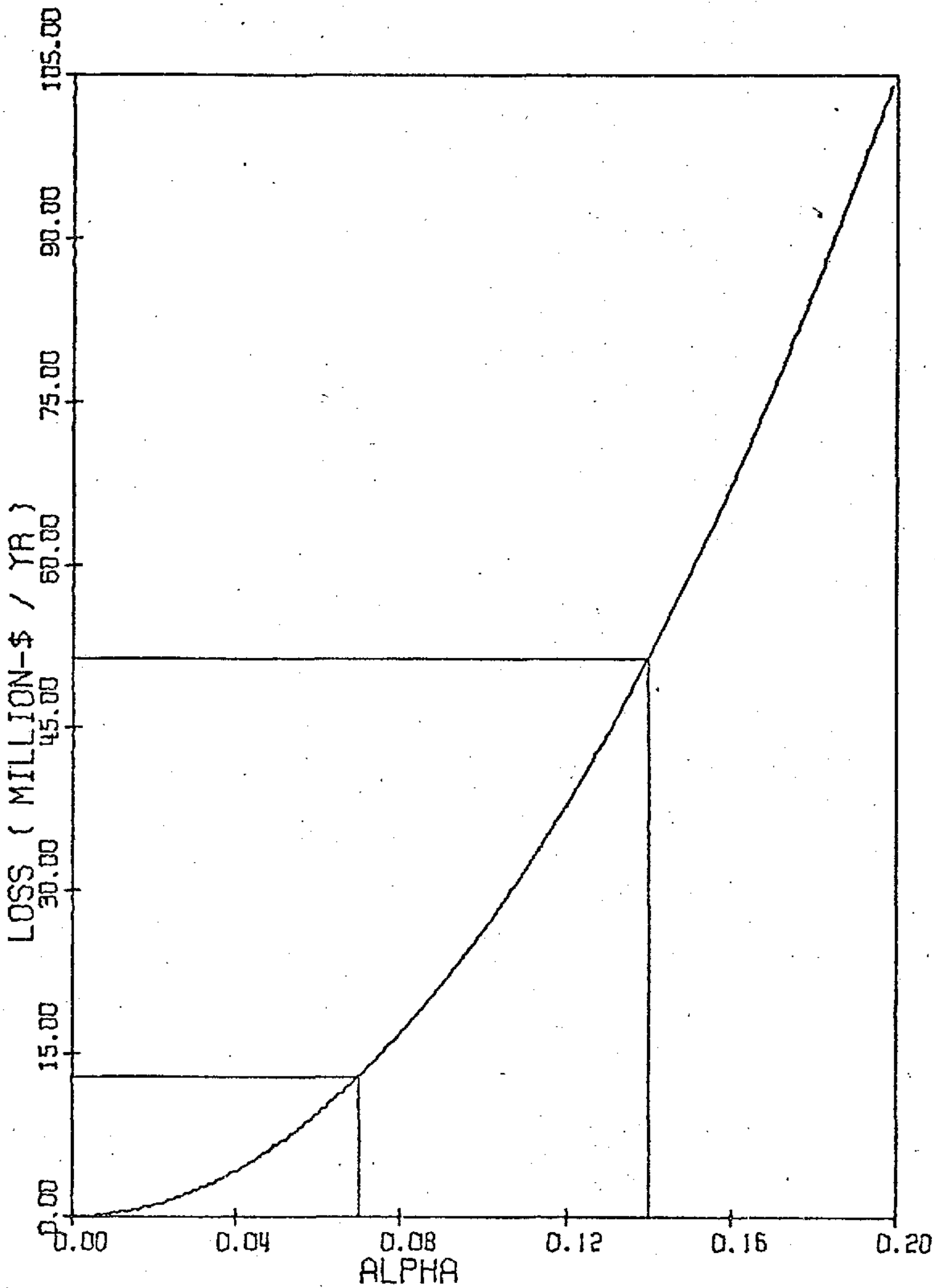

Figure 6.2: Annual Loss When Measurement Percentage Standard Deviation is Alpha/2, AL = 0, and Exports Absorb 42.5% of Harvest Variation

statistics on this subject are not available in the sources we have seen. Available studies, such as that by Gunnelson, Dobson and Pamperin* tend to focus on forecast error, which is a compound of Nature's variance and variance introduced by the measurement system. Statistics on forecast error contain, of course, some information constraining measurement error, but drawing implications from them requires very strong assumptions as to the underlying model. For our purposes these data are not suitable.

In their study of the value of improved statistical reporting, Hayami and Peterson encountered much the same sort of problem.** In their Table 1 (Ibid, p. 125) they present data on "typical sampling error" in major U.S. farm commodities prepared by the Statistical Reporting Service, U.S. Department of Agriculture. The methods by which the U.S.D.A. calculated these statistics are not specified, nor are definitions of the usual sort provided. By making some assumptions, however, we can use these data as the basis for plausible illustrative values in exploring our own results. Again, we would stress that these figures should be regarded as far from well established.

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* Gunnelson, G., W.D. Dobson and S. Pamperin, "Analysis of the Accuracy of USDA Forecasts," American Journal of Agricultural Economics, November, 1972, pp. 639-645.

**Hayami, Yujiro, and Willis Peterson, "Social Returns to Public Information Services: Statistical Reporting of U.S. Farm Commodities," American Economic Review, March 1972, pp. 119-130.

According to Hayami-Peterson, the U.S.D.A. as of the time of their writing conducted their surveys with a goal of attaining an average sampling error of 2 percent. Hayami-Peterson Table 1 indicates that this overall average performance corresponds to a sampling error of 2.1 percent for wheat. The error presumably refers to annual harvests, and we may regard it as applying to a sum of twelve monthly harvests. Denote by μ the error in measuring the annual harvest, AH, and by μ_i the error in measuring H_i, the ideal forecast of the harvest in month i. Using "hats" to denote measured quantities we have

$$(6.8) \qquad \begin{aligned} \hat{AH} &= AH + \mu \\ \hat{H}_i &= H_i + \mu_i \\ \hat{AH} &= \sum_{i=1}^{12} \hat{H}_i \end{aligned}$$

implying, if the measurement errors are independent,

$$(6.9) \qquad \sigma_\mu^2 = \sum_{i=1}^{12} \sigma_{\mu_i}^2 \quad .$$

By our assumption,

$$(6.10) \qquad \sigma_{\mu_i}^2 = \left(\frac{\alpha \bar{H}_i}{1.96}\right)^2 \quad .$$

6.26

Interpreting "average sample error" as the ratio of the standard deviation of μ to AH, we have, from Hayami-Peterson

$$(6.11) \qquad \sigma_\mu^2 = (2.1\ AH)^2 \quad .$$

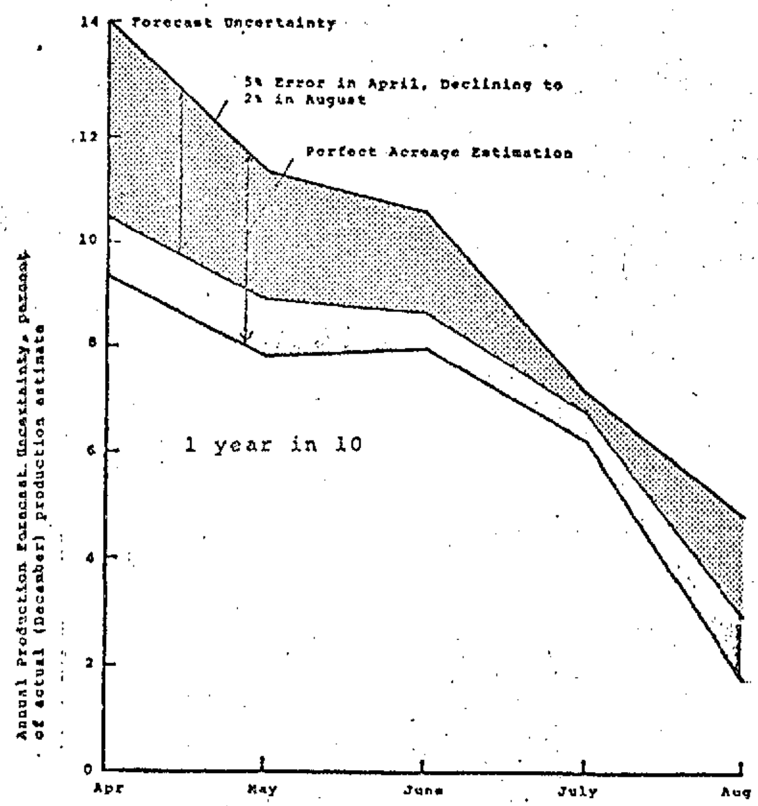
Substituting into (6.9), we have

$$(6.12) \qquad \alpha = \frac{(1.96)(2.1)}{\sqrt{\sum_{i=1}^{12} h_i^2}} \quad ,$$

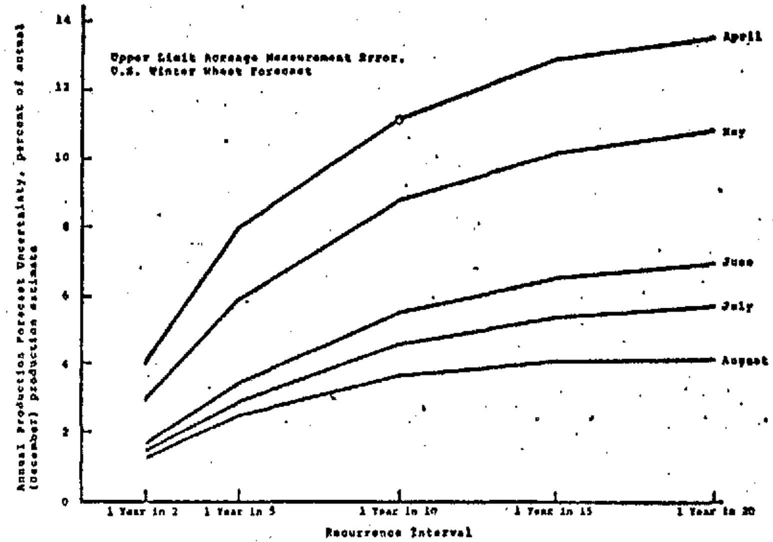
where h_i refers to the fraction of the annual crop harvested in the i^{th} month. Using the percentage distribution of the wheat harvest as described previously, the value of α can be calculated to be given by

$$(6.13) \qquad \hat{\alpha} = \frac{(1.96)(2.1)}{\sqrt{(.2964)}} = 7.559 \approx 7.6\% \ .$$

The Task Force on Agricultural Forecasting at Goddard attempts to assess likely improvements of ERS systems in forecasts of annual crops in perspective to present USDA performance. The results of the Task Force evaluation of likely improvements by our ERS system is shown, graphically, in Figure 6.3. Based on those results we may use the likely improvement in measurement by 50% as a convenient basis



Source: D. B. Wood, et. al., "The Use of the Earth Resources Satellite (ERTS) for Crop Production Forecasts", Draft Final Report, Goddard Space Flight Center, July 24, 1974



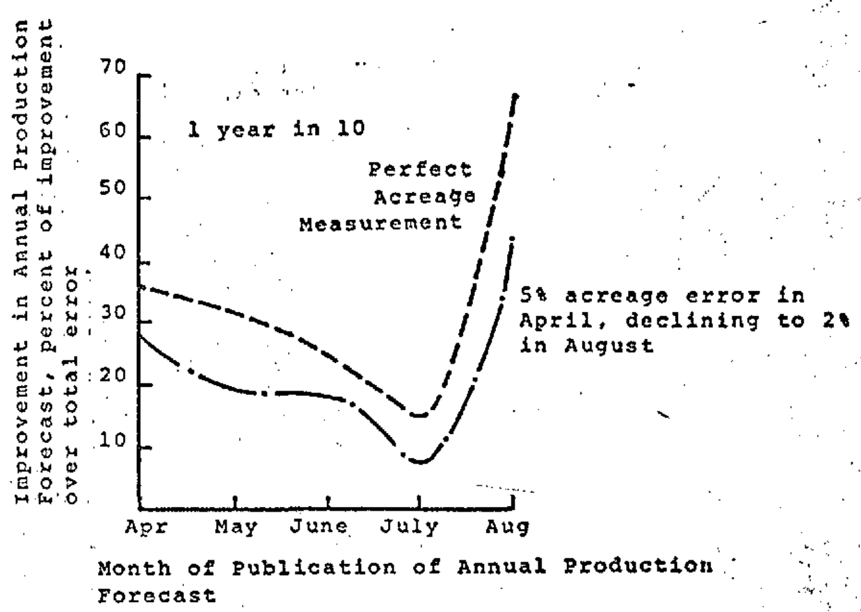


Figure 6.3 Contribution of Acreage Measurement to Improvement of Crop Forecast Accuracy

Table 6.5 The Value of Reducing Measurement Error Based on Goddard Task Force Results on ERTS (million of 4th qtr. 1973 dollars annually)

Price Elasticity for Wheat Demand [a]	α,95% confidence limit for percentage error in montnly harvest measurement			
	6%	7.92% [g]	10%	15.84% [h]
1. -.065 [b]	54.6	95.1	151.7	380.4
2. -.10 [c]	35.5	61.8	98.6	247.2
3. -.25 [d]	14.2	24.8	39.5	99.2
4. -.50 [e]	7.1	12.4	19.7	49.6
5. -0.75 [f]	4.8	8.2	13.2	32.8

a. United States domestic demand for all wheat, except as noted.
b. The authors of this report have estimated this value for "human purposes" (food) elasticity of demand for wheat.
c. EarthSat estimate in recent report to U.S. Dept. of the Interior
d. 50% reduction in the basic estimate, No. 4: for sensitivity analysis.
e. The basic estimate obtained by the authors for the price elasticity of unconditional demand for wheat (1971 data)
f. 50% increase in the basic estimate, No. 4: for sensitivity analysis.
g. α derived from 2.2% error in annual harvest (May crop measurement error for Winter Wheat).
h. α derived from 4.4% error in annual harvest (September crop measurement error for Spring Wheat).

for sensitivity analysis of the results. Table 6.5 gives the value of 50% improvement (not including cost savings by USDA if new methods are introduced and, of course, not netting out additional measurement costs) under a variety of changes in the parameters of the model.

The results described in Figure 6.2 and Table 6.5 indicate both the possibility of very substantial gains from reducing measurement errors in the crop forecasting system and the extreme sensitivity of the results to the values of current and potential measurement error variances.

Even relatively conservative assumptions (zero population growth, better current measurement, smaller percentage gain in accuracy) seem to suggest a rather substantial potential for gain from improved measurement accuracy. However, the great sensitivity of the results to variations in percentage accuracy, indicate that to obtain reliable estimates an effort must be made to discover more about current and potential measurement error.

At the same time the results described should make us sanguine about extending the measurements to other crops. The procedures generalize without any difficulty, and there is no obvious impediment to obtaining reasonably accurate measurements of all of the important parameters, with the exception, again, of the distributions of errors of measurement.

7. Concluding Remarks

All of the calculations in section 6 were directed toward evaluating a reduction in measurement error. However, as our discussion of forecasting in general in section 2 makes clear, the timeliness of information also importantly affects its value. This would be expressed in our model as reduced availability lag. This is an area in which satellite technology clearly promises substantial improvement, and it is one which may even have the potential for more substantial gains than found for measurement error reduction. Our estimates suggest rather substantial month to month variability in ideal forecasts, Nature's randomness. By reducing the availability lag by one month, we, in effect, eliminate one month's worth of variance. The value of this should be comparable to that of a similar reduction of variance due to measurement error improvement.

The components of this calculation are much the same as those assembled in Section 6. However, the formulae are more complex, owing to certain interactions among terms which take place when variance is reduced in this way. Programming and carring out these calculations should be a high priority follow-up research item.

Other extensions of the research are suggested by a review of the results described in section 6, which come at the end of a long and complex chain of reasoning and calculation. It is appropriate at the end of this report, then, to consider once again in summary fashion the links of the chain, to assess their strength, and to indicate how new ones can be added.

The basic logic of the model is simpler than its many details may lead one to believe. Grain production is taken to be exogenously given, but subject to random shocks obeying a (possibly complex) stationary stochastic law. Production in any period can be allocated to consumption (including use in the production of other goods) or additions to inventory. Inventories are determined by profit-seeking competitive agents, who base their decisions on forecasts of forthcoming grain harvests. In order to determine their current inventory levels, these agents must anticipate the future inventory levels as well as future harvests. They do this by assuming that all inventory holders understand the underlying demand and marginal storage cost relationships, and hence they in effect look for a market clearing set of spot and futures prices.

Given these facts, and having equipped ourselves with knowledge of the demand and marginal storage cost functions, we can describe the functional dependence of

inventory decisions produced by the market system and forecast harvests. This being the case, we can determine the relationship between measurement errors, as leading to forecast errors, and the average amount of variability to be expected in the grain consumption flow. Variability is a source of disutility -- marginal quantities of grain are more highly valued when consumption levels are low than when they are high, as reflected in the demand curve. Hence we can calculate the loss in value due to measurement error, and the gain due to its amelioration.

The weakest links in this chain are probably the early ones, for example, the very first one, which assumes grain production is exogenously given. We have argued in the text that a good case can be made for taking this assumption as a working hypothesis. Nevertheless, we should expect the results to be altered by the introduction of an endogenous production decision model of farmer behavior. That smoothing out of consumption and hence price movements over time is likely to have value to farmers should be obvious, given the history of the search for farm price stability.

The second link, shows a related weakness, in leaving out a set of decision makers. It was noted in the text that production is allocated not simply to consumption and inventory changes, but also to net exports, and in fact,

the empirical parameters of a very simple model of export determination importantly influenced the numerical results, as summarized in Figure 6.2 and Table 6.5. A final important group of agents is omitted at the third link at which it is assumed that grain inventories are determined by private entrepreneurs. In fact, certainly in the United States over the past twenty years, the government has been a major agency determining the quantity of grain in inventory.

How greatly the absense of these decision agents from the model affects the results is difficult to say. Surely, leaving out the dependence of production on prices causes our procedures to understate the value of improved information. On the other hand, the fact that farmers must make their planting decisions several months before harvesting leads us to guess that the additional benefit which will be found upon incorporating production to the model will be small relative to that attributed here to improved inventory decisions.

The direction in which the results are biased by our naive treatment of the export sector appears indeterminate. One could estimate the gain to the rest of the world attributable to improved inventory choices in the United States alone, and this would be expected to add to the total benefit. On the other hand, the extent to which the export sector acts to dampen the variance of domestic

consumption arising from variance in domestic production is too cursorily treated here to give a reliable indication of the results of a more careful study. Perhaps more important than these effects will be the consequences of more accurate forecasting of world-wide production. Since net exports can be treated as negative harvests in the U.S., and since world production will greatly influence net exports, the ability to predict world production has implications for even domestic inventory allocation improvement much like those studied here. (A whole-world model, on the other hand, is in principle simpler again, since there are no net exports.)

The policy of the U.S. government was, at least in large measure, directed toward price stabilization of grains over the past three or four decades. Insofar as the government is completely successful in this effort, the role of the private inventory holder is superceded, and speculative inventories will not be held. This would clearly affect the analysis in a major way, presumably in the direction of reducing the value of improved information, except, perhaps, as it determines the government's decisions. The most recent experience, of high grain prices, has temporarily, at least, taken the government out of the grain inventory business, and the broad outlines of the competitive model appear to hold.

The general way in which these three additional groups of agents can be systematically incorporated to the model is suggested by the accounting identity (7.1),

$$(7.1) \qquad C_t = (1-\delta)(Q^p_{t-1} + Q^g_{t-1}) + G_t - EX_t - (Q^p_t + Q^g_t) \ ,$$

where Q^p, Q^g, G, and EX, stand for, respectively, private inventory holdings, government inventory holdings, farm production, and net exports. Once these are determined, so is consumption, and hence benefit level. While the difficulties are likely to be somewhat greater than those encountered in this study, it would be interesting and useful to attempt to relate the decisions of the three new agents to the accuracy and timeliness of information for crop forecasting.

Extending the model to production decisions by competitive farmers is not likely to involve more than complication in the form of higher order difference equations, etc.. While the computational problems this can pose can be formidable, we would not anticipate major theoretical difficulties. The more challenging task is incorporating government and export sectors, particularly the former. The problems one can anticipate in the case of international demand are partly, again, those of sorting out the interactions of competitive producers and inventory holders. The

behavior of governments enters in the determination of international movements of grain (as the famous Russian wheat deal made abundantly clear), as well as into the nominally "government" sphere already alluded to, and it is in modeling the behavior of the important political actors, including the major agencies, that exceedingly interesting and possibly intractable problems lie.

Appendix A

Basic Data Sources

1) Chicago Board of Trade, Statistical Annual (1956 - 1972) Henceforth SA.

2) Federal Reserve Board, Federal Reserve Bulletin (March 1963, February 1965, March 1966, March 1967) Henceforth FRB.

3) --- Business Statistics (1971, 1973) Henceforth BS.

4) --- and the Massachusetts Institute of Technology, Quarterly Econometric Model (January, 1973) Henceforth FMP.

5) U.S. Department of Agriculture, Economic Research Service. Feed Statistics (September, 1967) and Supplement for 1971 (July, 1972) Henceforth FS.

6) --- Food Grain Statistics Henceforth FGS.

7) --- Supplement to Food Grain Statistics (1971) Henceforth SFGS.

8) --- Wheat Situation (May, 1973) Henceforth WS.

9) U.S. Department of Agriculture Statistical Reporting Service, Statistical Bulletin 277 (January, 1961), 387 (January, 1967), and 503 (December, 1972) Henceforth SB.

10) --- Cattle on Feed (January, 1973 and January, 1974) Henceforth COF.

a) Quantities

Visible Supply of Grains (millions of bushels) Monthly: SA

Total Stocks of Grains (millions of bushels) Quarterly: SA

Domestic Disappearances of Corn, Grain Sorghum, Oats, and Barley (millions of bushels) Quarterly: FS

Total Domestic Wheat Disappearance (millions of bushels)
1.) July 1964 - June 1970, Quarterly: WS.
2.) July 1955 - June 1963, Semi-annual: FGS

Food and Industrial Disappearance of Wheat (millions of bushels)
1.) July 1964 - June 1970, Quarterly: WS.
2.) July 1955 - June 1963, Semi-annual: FGS

Total Domestic Rye Disappearance (thousands of bushels)
1.) July 1966 - June 1971, Quarterly: SFGS.
2.) July 1955 - June 1966, Semi-annual: FGS

Cattle and Calfs on Feed in the states of Ohio, Indiana, Illinois, Minnesota, Iowa, Missouri, South Dakota, Nebraska, Kansas, Texas, Colorado, Arizona, and California (thousands of head) Quarterly: SB and COF

b) Prices

High and Low Futures Prices (pennies) Monthly: SA

Average price per bushel of number three barley at Minneapolis (dollars) Monthly: FS

Average price per bushel of number two white oats at Minneapolis (dollars) Monthly: FS

Average price per bushel of number three yellow corn at Chicago (dollars) Monthly: FS

Average price per hundred pounds of number two yellow grain sorghum at Kansas City (dollars) Monthly: FS

Average price per bushel of wheat at the farm (dollars) Monthly: SFGS

Average price per bushel of number two rye in Minneapolis (dollars) Monthly: SFGS and FGS

c) Other

Open market rate for prime commercial paper, 4 to 6 months duration (points) Monthly: FRB and BS

Gross national product (billions of dollars)
Quarterly: FMP

Unemployment rate (points) Quarterly: FMP

Consumer price index (1958 = 1.) Quarterly FMP

Population of the U.S. (millions of persons)
Quarterly: FMP

Consumer Price index (1967 = 100.) Monthly: BS